

The Use of Belief Revision to Model  
Contradictions and Entrenched Misconceptions

Normaziah A. Aziz



Ph.D.  
University of Edinburgh  
1996



## Abstract

The goal of this research is to illustrate the potential of belief revision in a tutoring system. The application is in a domain where contradictions arise when learners commit errors, and where misconceptions leading to these errors are deeply entrenched in the learners' reasoning process. We show how a belief revision system which employs the notion of *epistemic entrenchment* in resolving conflict can be useful in modelling learners and for remediation.

Our particular focus is on:

1. the domain of translating simple relational algebra word problems to algebraic equations;
2. misconceptions related to the *reversal error* i.e formulation of a *reversed equation*;
3. learner being persistent in holding the misconceptions which are referred to as the learner's entrenched misconceptions.

Our work also offers a possible learner modelling technique for contradictions and entrenched misconceptions — diagnosing through contradiction of beliefs and assisting the remediation process by using the beliefs and justifications involved when resolving the contradictions diagnosed.

We illustrate such investigations in TRAPS (*Translating Relational Algebraic Problems System*). TRAPS, a prototype computer-based tutoring system has been developed with a belief revision system embedded in its learner model. The belief revision system assists TRAPS in modelling and mending a learner's misconceptions. It also helps TRAPS in making overall observations of the learner's performance.



## Acknowledgements

I would like to thank the Malaysian Institute of Microelectronic Systems (MIMOS) for granting the financial support under award number (PPTP-ADB)282/92 which allows me to carry out the research described in this thesis.

I am also indebted to my supervisors Dr. Helen Pain and Dr. Paul Brna for their guidance, directions, constructive comments, support and encouragement throughout my work for this research.

I would like to thank my colleagues in the Dept. of Artificial Intelligence, in particular the AI-ED group — Susan Bull, Richard Cox, Judith Good and several others for their helpful pointers, suggestions and all the formal and informal discussion we had. To the academic and supporting staffs in the department, many thanks for making this department a conducive place for me to carry out this research.

I am grateful to all the students, adults and teachers who have been involved in the survey and evaluation exercises. Their participation is very much appreciated.

My special thanks goes to my most beloved mum and to my dad who had passed away a few months back. This thesis is specially dedicated to them, for their endless love, encouragement and many positive aspects of parenting. The family name in this thesis and previous publications based on the present work is a small token of my gratitude to my parents. I am also grateful for all the support and encouragement I received from my brothers and sisters.

## Declaration

I hereby declare that I composed this thesis entirely myself and that it describes my own research.

Normaziah A. Aziz



# Contents

<b>Abstract</b>	<b>ii</b>
<b>Acknowledgements</b>	<b>iii</b>
<b>Declaration</b>	<b>iv</b>
<b>List of Figures</b>	<b>xi</b>
<b>List of Tables</b>	<b>xii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Computer-based learning systems and learner modelling . . . . .	1
1.2 Handling contradictions and entrenched misconceptions . . . . .	3
1.2.1 Some terminology . . . . .	4
1.3 Motivation for this thesis . . . . .	5
1.4 Aims of this research . . . . .	6
1.5 Research described in this thesis . . . . .	7
1.6 Structure of the thesis . . . . .	8
<b>2 Learner modelling and belief revision</b>	<b>10</b>
2.1 Learner modelling . . . . .	10
2.2 Techniques in learner modelling . . . . .	12
2.3 Issues in learner modelling . . . . .	13
2.4 Inconsistency in learner modelling . . . . .	16
2.5 Belief revision applied in learner modelling . . . . .	17
2.6 Conclusion . . . . .	19

<b>3</b>	<b>Algebra word problems</b>	<b>21</b>
3.1	What is an algebra word problem? . . . . .	21
3.2	Norms and structure of algebra word problems . . . . .	22
3.3	Relational algebra word problems . . . . .	25
3.3.1	Empirical studies . . . . .	25
3.3.2	A further survey . . . . .	26
3.4	Errors of relational propositions . . . . .	27
3.4.1	Emphasis on the reversal error . . . . .	29
3.5	Solving algebra word problem . . . . .	30
3.6	The underlying cognitive process . . . . .	31
3.7	Diagnosing the problem solvers' conceptual understanding . . . . .	35
3.7.1	Identifying incorrect conceptual understanding . . . . .	35
3.7.2	Judging correct conceptual understanding . . . . .	37
3.8	Assisting the problem solvers . . . . .	37
3.9	Existing research in word problems . . . . .	37
3.10	Conclusion . . . . .	40
<b>4</b>	<b>The Belief Revision System</b>	<b>41</b>
4.1	The two belief revision systems . . . . .	41
4.2	Foundational and coherence approaches . . . . .	42
4.3	BRS' formalism . . . . .	45
4.3.1	Basic principles . . . . .	45
4.3.2	Epistemic entrenchment . . . . .	47
4.4	AGM postulates in BRS . . . . .	48
4.5	Reason Maintenance System in BRS . . . . .	48
4.6	Beliefs and inference representation . . . . .	50
4.6.1	The object level . . . . .	51
4.6.2	The meta level . . . . .	51
4.7	The Belief Revision System's features . . . . .	55
4.8	BRILM - the belief revision in TRAPS' learner modelling . . . . .	56
4.9	Some limitations of the revision systems . . . . .	58

4.10 Conclusion . . . . .	59
<b>5 Handling entrenched misconceptions and contradictions</b>	<b>61</b>
5.1 Dealing with entrenched beliefs . . . . .	62
5.2 Instructional design in TRAPS . . . . .	65
5.3 Characteristics of the contradictions . . . . .	68
5.4 Modelling through cognitive conflict . . . . .	72
5.5 Modelling the reversal error . . . . .	73
5.5.1 Diagnosing the misconceptions . . . . .	74
5.5.2 Remediating the misconceptions . . . . .	75
5.6 Modelling the observation conflict . . . . .	76
5.7 Conclusion . . . . .	77
<b>6 Implementation</b>	<b>80</b>
6.1 Stages of implementation . . . . .	80
6.2 Interacting with TRAPS . . . . .	81
6.2.1 Pre-test session . . . . .	81
6.2.2 Post-test session . . . . .	82
6.2.3 Teaching and learning session . . . . .	82
6.3 Pre-defined question library . . . . .	84
6.3.1 The Question library and the Question node . . . . .	85
6.4 Generating beliefs . . . . .	88
6.4.1 The representation of belief . . . . .	88
6.4.2 Sentence parser . . . . .	89
6.4.3 The learner's answers . . . . .	92
6.4.4 Implication of the learner's answers . . . . .	92
6.5 Adding beliefs to BRILM . . . . .	93
6.6 Handling contradiction of beliefs . . . . .	98
6.7 Explanation generator . . . . .	100
6.7.1 The Learner-TRAPS-beliefs explanation . . . . .	101
6.7.2 The Partial Canned Text explanation . . . . .	101

6.8	The learner's history record generator . . . . .	105
6.9	Conclusion . . . . .	108
<b>7</b>	<b>Evaluation</b>	<b>109</b>
7.1	Categories of evaluation . . . . .	109
7.2	Formative-internal evaluation . . . . .	110
7.2.1	Methods and outcome of this evaluation . . . . .	111
7.3	Formative-external evaluation . . . . .	111
7.3.1	Forms of formative-external evaluation . . . . .	113
7.4	First formative-external evaluation - Study 1 and 2 . . . . .	113
7.4.1	Subjects . . . . .	114
7.4.2	Material . . . . .	114
7.4.3	Methodology . . . . .	115
7.4.4	Observation and results . . . . .	117
7.5	Second formative-external evaluation - Study 3 . . . . .	122
7.5.1	Subjects . . . . .	122
7.5.2	Material . . . . .	122
7.5.3	Methodology . . . . .	123
7.5.4	Results . . . . .	125
7.6	Summary of TRAPS' evaluation . . . . .	129
<b>8</b>	<b>Discussion and Further Work</b>	<b>130</b>
8.1	Comparison with other related work . . . . .	130
8.1.1	On contradictions . . . . .	130
8.1.2	On entrenched misconceptions . . . . .	135
8.2	Analysis of contribution . . . . .	136
8.2.1	Modelling through cognitive conflict . . . . .	136
8.2.2	Explanation . . . . .	137
8.2.3	Computational Mathetics . . . . .	138
8.2.4	Other applications of learner modelling . . . . .	139
8.3	Limitations . . . . .	139

8.4	Directions for further research . . . . .	140
8.4.1	Tutoring strategy . . . . .	140
8.4.2	Other learning environments . . . . .	140
8.4.3	Other domains . . . . .	141
8.5	Conclusion . . . . .	142
<b>9</b>	<b>Conclusion</b>	<b>143</b>
	<b>Bibliography</b>	<b>147</b>
<b>A</b>	<b>A sample of TRAPS online session</b>	<b>154</b>
A.1	The Pre-test session . . . . .	154
A.2	The Teaching-Learning session . . . . .	157
A.3	The Post-test session . . . . .	163
<b>B</b>	<b>Questionnaire for survey</b>	<b>166</b>
<b>C</b>	<b>Questionnaire for Study 3</b>	<b>174</b>

# List of Figures

3.1	The <i>Time-Rate</i> family word problems. . . . .	24
3.2	Decomposition of the <i>Distance-Rate</i> word problem into a template. . . .	24
5.1	A sample of the diagnosing session in TRAPS. . . . .	64
5.2	Level-1 conflict - the <i>triangle conflict</i> that occurs when learner commits the reversal error. . . . .	69
5.3	Level 2 conflict - TRAPS's meta level observation on the learner's performance. . . . .	71
5.4	The learner modelling technique in TRAPS. . . . .	73
5.5	Diagnose and remediation in TRAPS' learner modelling. . . . .	74
5.6	Diagnosing a learner's errors and misconceptions within one problem set. .	75
6.1	An example of a TRAPS-learner dialogue during the <i>teaching and learning</i> session. . . . .	83
6.2	Each problem set is decomposed into several questions and arranged by their level of difficulty in a cascaded manner in the Question Library. . .	86
6.3	Several underlying concepts in this single question. . . . .	87
6.4	The standard template for the relational algebra word problem dealt with in TRAPS. . . . .	89
6.5	The grammar in TRAPS' DCG. . . . .	90
6.6	The vocabulary in TRAPS' DCG. . . . .	91
6.7	A question posed to the learner. The answer determines the input that will be generated as the learner's belief for this question. The belief to be generated from either of these answers is shown in the next figure. .	93
6.8	Based on the learner's answer, an appropriate learner's belief will be added to BRILM. . . . .	93

6.9	Each of the learner's answers has its own implication. Only existing beliefs (that have been added as the learner's belief) will have their implication belief generated and added as the Learner Model's belief. . .	94
6.10	Implication of formulating the reversed equation - causing a contradiction on the "qualitative value" belief proposition. . . . .	95
6.11	A sample of the diagnosing session with TRAPS. . . . .	96
6.12	Level 1 conflict has been detected by the Belief System and TRAPS is being notified. . . . .	96
6.13	Evaluating the question values for conflict between " <i>the learner is capable of translating relational algebra word problems</i> " and " <i>the learner is not capable of translating relational algebra word problems</i> ". . . . .	99
6.14	A triangle conflict had been diagnosed. The LTB-explanation is shown here as the remediation session for the learner. . . . .	102
6.15	Generating the Partial-Canned Text explanation for <i>static-comparison approach</i> . . . . .	103
6.16	A Partial Canned Text explanation for <i>static-comparison approach</i> incorrectly applied to the <i>playground problem</i> . . . . .	104
6.17	Displaying to the learner his incorrect answers. If he chooses to fix his answer, the related question will be posed. . . . .	106
6.18	A meta-level observation conflict and part of its explanation. This information is recorded in a separate file for the teacher's reference. Note: RAWP stands for Relational Algebra Word Problems. . . . .	107

# List of Tables

3.1	Common approaches that students apply in translating and formulating relational algebra word problems. . . . .	34
3.2	Problem solvers' misconceptions and missing concepts related to the reversal error. . . . .	35
3.3	Related works in the domain of word problems. . . . .	39
5.1	Instructional objectives and the related dialogue strategies (Halff, 1988). . . . .	66
7.1	Learners' performance when using TRAPS in Study 2 of the first formative-external evaluation. . . . .	119
7.2	Learners' feedback on the <i>LTB-explanation</i> and the <i>PCT-explanation</i> in Study 2 of the first formative-external evaluation. . . . .	121
7.3	Teachers' feedback on agreeing or disagreeing with the beliefs derived by TRAPS. . . . .	126
7.4	Teachers' feedback on rating the level of difficulty of some questions in TRAPS. . . . .	127
7.5	Teachers' feedback on using the <i>LTB-explanation</i> and the <i>PCT-explanation</i> in classes. This was conducted in Study 3. . . . .	128
8.1	Some issues considered in THEMIS and TRAPS. . . . .	131



# Chapter 1

## Introduction

### 1.1 Computer-based learning systems and learner modelling

Computer-based learning systems provide a one-to-one learning environment, where learners can proceed through the teaching material at their own pace. They can provide the opportunity for the learners to practice in particular areas of the subject in which they do not perform well. Such systems can function as a supplementary learning tool for the learners. The system can also act as an assistant to the teacher eg. by being able to offer extra one-to-one tuition which the teacher may not have time to provide in a real classroom.

However, for a learning system to embody a real educational value for learners and teachers, it has to be as knowledgeable and responsive to the learner's needs as a human teacher would be. Systems that are developed with the aim of incorporating the characteristic of being "*self-adaptive*" are known as *Intelligent Tutoring System* (ITS) or *Interactive Learning Environments* (ILE). In relation to this, there is a trend of migration in developing traditional computer-based learning systems (those that are not self-adaptive) to systems that are able to *adapt*<sup>1</sup> to their users'<sup>2</sup> needs. To achieve developing the latter computer-based learning systems, the need to model the user arises. Modelling the user is considered to be the solution to producing individualised instruction. In most Artificial Intelligence in Education (AI-ED) systems, the user

---

<sup>1</sup> Adapt to some extent that has been defined in the system's design.

<sup>2</sup> The learners and/or teachers.

(learner) model seems to be a central component that portrays the characteristic of intelligence and towards individualisation in the system.

- purpose repeated p13

However, modelling the user is a dynamic process and not easy. It depends on the user's behaviour, some of which is predictable but some is unpredictable. Behaviour in this context is defined by (Dillenbourg & Self, 1992) as "*a sequence of actions performed by the user as a potential solution for some problem*". For the purpose of discussion in this section, we limit our discussion of users to the learners. Hence, the user modelling is referred to as the learner modelling. Learner modelling is the process by which a computer-based learning system develops and maintains an understanding of the learner who is using the system.

As highlighted by Self:

*"(Learner) modelling was once considered to be the key component in providing individualized instruction, and a number of useful techniques were developed. Then, as the intrinsic difficulty of the task became apparent and as more constructivist philosophies were adopted, (learner) modelling was largely abandoned. Now, however the tide is turning again. Three recent developments suggest an increased and important role for (learner) models: first, ... the prospect of general-purpose learner modelling shells; second, ... the use of multi-agent techniques from artificial intelligence for (learner) modelling; and third, ... presentation (in multimedia) needs to adapt to individual learners."*

(Self, 1995), p. 40.

This thesis relates to the second category of recent developments that Self describes above. The multi-agent environment we deal with is the interaction and exchange of beliefs and knowledge between the learner and the system (that acts as a tutor). A more complex and dynamic multi-agent environment<sup>3</sup> may occur in a collaborative computer-based learning system. In the research described in this thesis, we focus on the inconsistency of beliefs or information throughout the learner-system interactions. Detecting and resolving inconsistency, i.e contradicting beliefs, is performed by an

---

<sup>3</sup> Learner-Tutor, Learner1-Learner2, Learners-Tutor etc. types of interaction

Artificial Intelligence (AI) technique, the **belief revision system**. In doing so for a particular subject domain i.e **translating relational algebra word problems into equations**, we detect and resolve not only contradicting beliefs between the learner and the system, but also within the learner’s own beliefs, and among the system’s own beliefs also.

## 1.2 Handling contradictions and entrenched misconceptions

Both the learner and the system (tutor), as epistemic agents hold their own set of beliefs and knowledge about the world. As new evidence about the world being learned, there may occur a time when this new evidence contradicts or extends their existing belief set. When this new information creates an inconsistency with prior beliefs, a *revision* of the belief set has to be made. This revision involves identifying which of the old and new beliefs conflict and caused the inconsistency, and deciding whether to accept the new information or to eliminate the old.

However, people are in general very reluctant to change their current beliefs sets. They usually reject, or ignore or reinterpret the new information which contradicts with their current beliefs. They seldom attempt to add it to their beliefs and make necessary adjustment to their prior beliefs. Such a situation is true for people encountering either correct or incorrect new contradicting beliefs.<sup>4</sup> We regard such behaviour as having entrenched beliefs.<sup>5</sup>

In relation to the description of contradictions, revision of beliefs and entrenched beliefs, these issues appear in several domain of mathematics and science. In this thesis, we focus on translating relational algebra word problems into equations. These issues are found to be relevant when learners commit the reversal error.<sup>6</sup> We propose and develop a possible technique to support modelling in such a domain — the use of *belief revision systems with the notion of epistemic entrenchment*. We illustrate such investigations in TRAPS (*Translating Relational Algebraic Problems System*). TRAPS,

---

<sup>4</sup> This will be discussed more in section 5.1.

<sup>5</sup> Referring to their prior or existing beliefs.

<sup>6</sup> This will be defined in section 1.3

a prototype computer-based tutoring system has a belief revision system embedded in it which assists it in modelling and mending a learner's misconceptions.

### 1.2.1 Some terminology

This section will clarify some of the terminology and standards that are used throughout this thesis.

- In previous publications (Aziz, Pain, & Brna, 1995a) and (Aziz, Pain, & Brna, 1995b), the prototype system we developed in this research is referred to as TAPS (Translating Algebraic Problems System). However, TAPS is coincidentally similar in name and domain to (Derry & Hawkes, 1993)'s system, although the two systems evolved completely independently of each other. In order to avoid confusion, we have changed the name of our system to TRAPS (Translating Relational Algebraic Problems System), which focuses on the exact algebra domain we are dealing with and describes one of our modelling technique's aims i.e entrapping contradictions and entrenched misconceptions.
- A **reason maintenance system** is a system that maintains consistency of beliefs or information in the light of new evidence. In this thesis, we define consistency of beliefs as not having contradicting beliefs  $B$  and  $\neg B$  at the same time, in the same database. Maintaining the consistency of beliefs involves detecting the occurrence of contradiction of beliefs and resolving the conflict by revising the beliefs and justifications involved in the conflict. Such a maintenance process is also referred to as **belief revision**, and the system that performs it as a **belief revision system**. Throughout this thesis, the term belief revision and belief revision system will be used in our discussion.
- The term **contradiction** of beliefs and **conflict** of beliefs are regarded as synonymous in this thesis.
- The term for the area of Artificial Intelligence in Education will be abbreviated and referred to as AI-ED throughout the discussion in this thesis.

- Student and learner are synonymous in some contexts. However, the term learner is preferred in this thesis, to emphasize the notion of learning for any user (a student or non-student). Hence, the modelling process for the user (learner) in this research is referred to as **learner modelling** instead of student modelling. User modelling will be used when we refer to modelling systems other than ITS or ILE.
- For consistency, all learners are described as *he* in our discussion except for the female students of our experiment in chapter 7 who are described as *she* in our description of observations on them.

### 1.3 Motivation for this thesis

Several empirical studies have observed that many students face difficulties in translating algebra word problems with sentences that state relationships between two variables, e.g. (Mayer, 1982) and (Paige & Simon, 1966). Among the typical errors that are often made when translating such problems is formulating a *reversed equation*, i.e. an incorrect equation that produced a reversed implication (or meaning) of a correct equation. As an example<sup>7</sup>, if  $3C = T$  is the correct equation for a given relational algebra word problem, the reversed equation is  $3T = C$  or  $C = 3T$ . This error is referred to as the *reversal error*. It has been observed that some of the misconceptions which lead to the reversal error are caused by inconsistency and contradiction of their beliefs in reasoning.

The learner's misconceptions related to the reversal error are usually deeply seated and difficult to dislodge (Lochhead & Mestre, 1988). These misconceptions have been found to be widespread across academic background, age, and nationality (Clement, 1982); (Schoenfeld, 1985) and (Mestre & Lochhead, 1983). A further survey<sup>8</sup> of adults translating algebraic problems (words, tables and picture) into algebraic equation carried out by the author supports this.

The skill of translation is essential and crucial but teachers do not have much time

---

<sup>7</sup> A more detail description and examples will be discussed in chapter 3.

<sup>8</sup> This will be discussed in section 3.3.2.

to spend on it or to give full attention and assistance to students during the limited hours in class. There is a need to provide learners with additional support in the form of an *intelligent* computer-based tutoring system, that will allow them to practice the translation process and hence improve their skills in translating relational problems to algebraic equations.

The outstanding characteristics related to the reversal error are **the occurrence of contradiction of beliefs** and that **misconceptions** leading to this error are **entrenched** in the learner's cognitive reasoning. In the area of AI, work<sup>9</sup> on belief revision deals with contradictions of information or beliefs in the light of new evidence. A belief revision system detects and resolves the contradiction between beliefs to maintain the consistency of beliefs in a knowledge base. It is strongly felt that a relation can be established between **belief revision and learner modelling** to *model for remediation* learners who have difficulties with the reversal error. The author is motivated to integrate a belief revision system in the learner modelling for a tutoring system that deals with contradictions and entrenched misconceptions. Schank's remark (as below) is another strong motivation for this research:

*"AI is a subject that is intimately bound up with education. AI concerns itself with getting machines to read, to reason, to express themselves, to make generalisation, and to learn. ... AI people are in a unique position to improve education because they are competent in the latest technology and concerned with the issues in learning and understanding."*

(Schank & Edelson, 1990), p.1

## 1.4 Aims of this research

The overall aim of the research is to apply a belief revision system as the basis for learner modelling in a domain with contradictions and entrenched misconceptions (in particular translating relational algebraic word problems) and explore the potential of such learner modelling.

---

<sup>9</sup> Theoretical and/or implementation



The detailed research aims are to:

1. Implement the combination of the *instructional design based on theories of how people deal with entrenched and contradicting beliefs* and the *belief revision system* to elicit learners' entrenched contradictory schemes<sup>10</sup>.
2. Demonstrate the usage of a belief revision system that resolves contradictions by the notion of epistemic entrenchment to gain consistency:
  - between the learner's beliefs and the system's beliefs;
  - within the learner's own beliefs;
  - among the system's own beliefs.
3. Demonstrate the role of the belief revision system in providing relevant beliefs and justifications as explanations to remediate the learners with conflicting beliefs.
4. Illustrate a prototype tutoring system that is intended to lessen (or remove) learners' conceptual difficulties that impede their skill in translating relational algebra word problems to mathematical symbols.

Later sections in this thesis will refer to these specific aims and attempt to establish how far they have been met.

## 1.5 Research described in this thesis

This thesis starts with the analysis of the domain i.e translating relational algebra word problems into equations. The analysis focuses on the reversal error, the compilation of misconceptions related to the reversal error, the study of characteristics of the contradictions occurring and the behaviour of learners who commit the reversal error. A learner modelling technique based on these findings has been design<sup>ed</sup> to tackle contradictions and entrenched misconceptions, the two major features of the reversal error. An instructional design that is to support our modelling technique has also been worked out.

---

<sup>10</sup> This will be defined in section 3.6.

Analysis of the domain is followed by evaluation and testing of a belief revision system (Berendt, 1992) that functions as a decision aid. The properties of Berendt's system and the results of our study on the reversal error in relational algebra word problems are combined and further analysed. Berendt's system has been modified in order to produce a new version of the belief revision system i.e *BRILM* (Belief Revision In Learner Modelling) that aims to assist our learner modelling technique.

Contents of the domain which includes questions, answers, correct concepts and misconceptions are developed as the domain, pedagogical and interface modules for a tutoring system. *BRILM* is invoked to model the learner and the system. The overall tutoring system is called *TRAPS*. Evaluation of *TRAPS* as a prototype tutoring system has been conducted as an assessment of our learner modelling approach and the potential of *BRILM* in *TRAPS*.

Detail of each research phase will be discussed in this thesis.

## 1.6 Structure of the thesis

The remaining chapters of this thesis will be structured as follows:

Chapter 2 describes the background of learner modelling and belief revision. A general description of the learner model, its functions, techniques used, and important issues related to learner modelling are discussed. These are then followed by highlighting the problem of inconsistency of information while modelling learners and the role of belief revision in such a problem.

Chapter 3 reviews the domain that we are dealing with, algebra word problems. We start off with the characteristics of algebra word problems and their norms and structure. Discussion then proceeds with relational type algebra word problems and errors related to them. Some cognitive phases and common strategies applied by problem solvers are described. This is then followed by a description of the problem solvers' performance that demonstrates their conceptual understanding. The chapter ends with some existing work that deals with word problems.

In chapter 4 discussion on the core of our learner modelling, i.e the belief revision



system is made. We introduce the two belief revision systems involved in our work — *Belief Revision System (BRS)* and *Belief Revision In Learner Modelling (BRILM)*. The former has been the basis for the latter system. Description of *BRS* in terms of its underlying theory, formalism, implementation and features are presented. These are then proceeded by describing the modification made to produce *BRILM*. *BRILM*'s capabilities and limitations are also highlighted.

Chapter 5 describes TRAPS' instructional design that is meant as an approach to elicit and handle learners' entrenched misconceptions. This is then followed by a description of the characteristic of contradictions in TRAPS — the Level-1 and Level-2 conflicts. The discussion proceeds with the description of how TRAPS model both levels of conflicts.

Chapter 6 discusses the implementation of TRAPS. The discussion is based on the components that TRAPS has and presented in the sequence in which they are implemented. We discuss the overall interaction session between TRAPS and the learner, the generation of beliefs, adding and updating them in *BRILM*, and our approach for handling contradicting beliefs in *BRILM*. Following this, the explanation generator and learner's record generator are described.

In chapter 7 we discuss the evaluation of TRAPS as a prototype system. Categories of evaluation in ITS in general are described at the beginning of this chapter. This is then followed by a short description of our formative-internal evaluation and formative-external evaluation. The stages, subjects, materials and methodologies involved in the latter evaluation are discussed. The observation and results of these stages of evaluation are also reported. Finally a summary of TRAPS's evaluation is discussed.

Chapter 8 begins with some comparison and similarities of other work that is closely related to ours. We then proceed with the analysis of the contribution that the research in this thesis has made, particularly in the AI-ED discipline. Directions for further or future related work are suggested at the end of this chapter.

Chapter 9 is the conclusion. The results are summarised. The relative success and the limitations of applying a belief revision system in learner modelling, for the domain where contradictions and entrenched misconceptions occur, are summarised.

## Chapter 2

# Learner modelling and belief revision

In this chapter, we discuss the ~~the~~ background of learner modelling and belief revision. A general description of the learner model, its functions, techniques used, and important issues related to learner modelling are discussed. These are then followed by highlighting the problem of inconsistency of information while modelling learners and the role of belief revision in such a problem.

### 2.1 Learner modelling

The user model is a component in a computer-based system that functions to adapt the instruction or flow of the system to the user's need. It does so by developing and maintaining an *on-line understanding* of the user while he uses the system. In systems that are designed for students or learners, the user model is referred to as the student model or learner model. Examples of such systems are ITS and ILE.

In the discussion of this thesis, we will use the term learner model. The process involved in a learner model is referred to as **learner modelling**. Learner modelling deals with dynamic environments particularly in relation to the learner's belief. We define a belief as information<sup>1</sup> believed to be true by the agent or believer. Modelling a learner does not limit its scope within the learner's belief alone. It involves other agents' beliefs also. Other agents could be the system, the tutor or other students involved (as in

---

<sup>1</sup> This will be discussed again in section 2.5.

statement

collaborative learning environment) during the on-line session.

Learner models can be categorised as static or dynamic models. A static model process is predetermined before the system is used. It cannot react to unanticipated learner's input. Time and effort have to be spent in developing the domain specific information about misconceptions. Examples of a static learner model is as those in WUSOR, a tutoring system designed to teach students playing the game WUMPUS (Goldstein, 1982). WUSOR's genetic graph which is used in its modelling is static, i.e it exists for the game before it is played and does not change. In contrast, a dynamic learner model can infer the learner's misconceptions from analysing the learner's behaviour, and hence can diagnose misconceptions underlying errors which have not been encountered before. Several dynamic learner models are as in Automated Cognitive Modelling (ACM) (Langley & Ohlsson, 1984) and *ArtCheck* (Sentance, 1993).

TRAPS' learner modelling is a mixture of a static and dynamic model. It is static in terms of its limitation on modelling anticipated and preset misconceptions only. It is dynamic in that the modelling may change its modelling results after being revised when a conflict of beliefs occur during the modelling process.

Several actions related to *teaching and learning* have been associated with learner modelling. These processes relate to the function of learner models. The <sup>uses</sup>usage of learner models<sup>2</sup> among others are:

<sup>805</sup>  
**diagnostic** - to help detecting errors and misconceptions related to the errors;

**correction** - to assist repairing incorrect concepts or knowledge;

**elaboration** - to help extending further explanation;

**prediction** - to assist determining the learner's likely responses to tutorial actions;

**planning** - to help guiding the pedagogical decision making i.e tutoring strategy;

**evaluation** - to assist assessing the learner's or the system's performance.

In our research, the scope of TRAPS' learner modelling touches the function of diagnosis, correction, elaboration, planning and evaluation. It *diagnoses for remediation*,

---

<sup>2</sup> This is also discussed as functions of the student model in (Self, 1988b).

where diagnose involves detecting an error, analysing the error, uncovering misconceptions which are the source of the error. On the other hand, remediation involves giving appropriate and helpful explanation to the learner to correct his misconceptions.

Our learner model, *models for remediation* which can also be described as defined in (Self, 1995):

#### DIAGNOSE & SYSTEM'S DESCRIPTION & OBSERVATION -> ACTION

For such a learner modelling goal, our tutoring system must have a means of recognising and representing the learner's error. Hence, TRAPS' (system) description and observation play an important role here. Correction and elaboration are the two processes that we regard as ACTIONS. Another process that falls under ACTION is planning for the next instruction or questions to proceed during the on-line session. An evaluation of the learner's overall performance is an overall observation made by TRAPS. Further description of all these processes will be discussed in chapter 5.

## 2.2 Techniques in learner modelling

Two well known learner modelling approaches that many educational computer-based systems are based on the <sup>age</sup> **overlay modelling** and **perturbation modelling** approaches. In overlay modelling, the learner's knowledge is represented as a subset of the expert knowledge. The model is maintained by the system marking which piece of information is known and not known by the learner. An example of classic overlay modelling is in GUIDON (Clancey, 1987), an ITS built to assist medical students learning from MYCIN, an expert system (Shortliffe, 1976). On the other hand, perturbation modelling involves developing a model of the learner which incorporates misconceptions held by the learner. Under this modelling approach, several techniques to assist diagnosing misconceptions such as the Bug library (Brown & Burton, 1978) and Repair theory (Brown & VanLehn, 1980) have been used. These techniques are illustrated in systems such as BUGGY (Brown & Burton, 1978), a system to assist student improve their arithmetic skills and DEBUGGY (Burton, 1982) which extends BUGGY into a diagnostic system.

??

In most AI-ED systems (ITSs or ILEs), the learner model is a central component that portrays the characteristic of *intelligence* and *towards individualisation* of the system. Within the AI-ED community, several techniques derived from some well-known AI techniques have been developed and applied in learner modelling. These techniques are sometimes applied as extension of either the overlay or perturbation approach to improve the modelling. The following are a list of some of AI techniques applied in learner modelling:

- Machine learning (Sentance, 1993), to dynamically generate mal-rules in response to student behaviour which are not pre-determined by the system designer. (Woolf & Murray, 1993), had also applied machine learning technique to account for inconsistency, spontaneity and forgetting issues when modelling a learner;
- Neural networks (Mengel & Lively, 1991), where the network is trained to simulate a student's cognitive processes;
- Fuzzy logic (Derry & Hawkes, 1993), to provide an approximate diagnosis, recognising that a student's behaviour is not entirely consistent and induction from it is risky;
- Bayesian belief networks (Katz, Lesgold, Eggen, & Gordin, 1992), to provide a less precision-oriented approach to learner modelling;
- Model-based diagnosis (Self, 1993), to cast the student modelling in the terms of general diagnosis in AI;
- Belief revision (Huang, 1993), (Kono, Ikeda, & Mizoguchi, 1994), (Paiva, Self, & Hartley, 1994) and (Aziz et al., 1995b) to maintain consistency of information when modelling the learner;

## 2.3 Issues in learner modelling

Learner modelling in a tutoring system provides information about what the learner understands and does not understand. The modelling process builds up and maintains



an understanding of the learner while he uses the system. The goal of the modelling is to enable the system to individualise accordingly to the learner's capability and needs. To achieve such a goal is not easy. It is a rather complicated process and many issues have to be dealt with. Among issues discussed by (Mizoguchi, 1993), three basic issues concerning learner modelling are:

1. Trade-off between accuracy of modelling data and computational time and cost.

The more accurate a learner model is, the more *individualisation* performance the system may offer. However, there is a trade-off between accuracy of the model and the cost to construct and maintain it. As highlighted by (Self, 1988a), from a pragmatic point of view, there ought to be an appropriate representation scheme for learner models by taking such a trade-off into consideration;

2. Maintaining consistency of information involved while modelling. Basically, the process involve in learner modelling works at least on a pair of information (data), i.e a problem and a solution of the problem. But multiple data that are involved in the modelling is common too. Hence inconsistency or contradiction of data may arise. For example, in reality, learners' beliefs may or may not be consistent with what is expected by the tutor. A contradiction can occur here. At the same time, while modelling a learner, the tutor's belief about the learner at one time may contradict its (tutor's) former belief.

3. Domain-independent theoretical foundation for learner modelling mechanisms.

Many existing learner modelling are domain-specific which requires a lot of domain-specific information. Hence, it is difficult to extract common properties of modelling methods to accumulate them as theoretical results.

Several works concentrating on tackling these issues have been going on in the AI-ED community. We describe some of these works.

Ohlsson's description and illustration on *constraint-based student modelling* (Ohlsson, 1993) offers a possible reduction of the expensive computation of learner modelling. Constraint-based modelling approach represents the subject matter of knowledge in sets of constraints. Constraint violations on the learner indicate incomplete or incorrect knowledge. Such a technique allows us to eliminate the need for runnable models of

either the expert or learner and to reduce the computation for learner modelling to pattern matching.

As for the second issue, i.e maintaining consistency of information in learner modelling, (Huang, 1993) and (Kono et al., 1994) are among few works that focus on handling inconsistency or contradictions of belief (or information) involved. Huang worked on the *logic of attention* to capture the learner's inconsistent beliefs and attention. This approach could be applied in a Socratic tutoring system. The logic applied in their reasoning system, called ABRS (Attention-shifting belief reasoning system) revises only beliefs that are under attention or being focused on during the modelling, and hence improve belief revision computational speed (Huang, McCalla, & Neufeld, 1991) which is another answer to the first learner modelling issue stated before. (Kono et al., 1994) have also developed *THEMIS*, a nonmonotonic induction student modelling which formulates contradictions that arise<sup>3</sup>. Related to this issue of contradiction of beliefs, our work as described in (Aziz et al., 1995a), (Aziz et al., 1995b) and discussed in detail in this thesis offers a possible learner modelling technique — diagnosing through contradiction of beliefs and assisting remediation process by using beliefs and justifications involved when resolving the contradiction diagnosed. More will be discussed in chapters 4, 5 and 6.

For the third issue, research such as that of (Dillenbourg & Self, 1992), (Self, 1992), (Paiva et al., 1994) and (Paiva, Self, & Hartley, 1995) are among significant works in leading to general-purpose learner modelling shells or learner modelling workbench. The first two works deal with formalising frameworks for learner modelling, while the third research develops a learner modelling shell. Such works may offer possible solution<sup>5</sup> for a domain-independent theoretical foundation in learner modelling.

Learner modelling in TRAPS deals in depth on the second issue and offers some prospects<sup>4</sup> for the third issue that Mizoguchi raised. The issue of trade-off between accuracy of modelling and computational time and cost is not feasible and beyond the scope of our research, currently. The main focus of our research is on the issue of handling contradictions to maintain consistency. This can be achieved by using a

<sup>3</sup> This will be described further in section 8.1.1

<sup>4</sup> This will be discussed in section 8.4.3

**belief revision system.** The next section discusses inconsistency when modelling users (learners).

## 2.4 Inconsistency in learner modelling

Among important issues of learner modelling as discussed by (Woelf & Murray, 1993) and (Mizoguchi, 1993) is the capability of handling the inconsistency of information. The inconsistency of information that we mean is **contradiction of information**, i.e. having both information  $b$  and  $\neg b$  at the same time in the same database.

(Cawsey, 1992) raises two issues and possible solutions adopted in the user modelling community, on representing uncertain information and deal with changing and conflicting assumptions:

1. What to do when there arises conflicting information concerning whether the user understands some concept? As an example, at one point the user's performance may suggest that he understands a concept, but at a later point of time his poor performance (or incorrect answer) suggests that he does not understand it.

Two common approaches that could be applied:

- (a) Rules for updating the user model are arranged in such a way that more certain rules are always considered before less certain ones. The less certain <sup>rules</sup> information are the ones ignored.
  - (b) Numerical values are assigned to rules and facts in the user model. Cawsey claims that this approach gives the opportunity to both the certain and uncertain information to contribute to the model. But the question is how to combine the different weights of value appropriately.
2. How to revise the remainder of the user model when part of it has been changed?  
A change in part of the user model may nullify other related parts of the model because inference rules may have used the old information to indirectly derive new assumptions.

Two methods of maintaining such a situation in user modelling are:



- (a) To use the inference rules by backward chaining when the system needs to know if the learner understands some concept. If the foundation assumptions about the learner change, derived assumptions will implicitly change. Such an approach is applied by (Finin, 1989) in his GUM system by using default logic to deal with conflicting inferences.
- (b) In inference rules that are applied, whenever new data is added to the model by forward chaining, justifications for facts in the learner model have to be maintained explicitly. In this situation, there is a need for a reason maintenance system to update the learner model. Such a method is used by (Murray, 1991), (Huang, 1993), (Kono et al., 1994), and (Paiva et al., 1994) in their learner model.

Learner modelling in TRAPS utilises a reason maintenance system<sup>5</sup> (as in 2(b)) and assigns numerical values to the facts and rules (as in 1(b)), as part of the mechanism to revise the beliefs. Detail technical description will be discussed in chapter 4.

A description of belief revision system and how it can be used in the context of learner modelling is given in the following section.

## 2.5 Belief revision applied in learner modelling

Learner modelling can be represented as changes of beliefs in the belief sets of the learner and the system (in particular those beliefs about the learner's beliefs) (Self, 1992). Such representation of learner modelling relates to some central concerns of formal AI i.e non-monotonic reasoning and belief revision.

In non-monotonic reasoning, new conclusions can invalidate information that is already known or has been deduced. Reason maintenance systems are applied to resolve the limitations of monotonic logic. **Belief revision** deals with inconsistency of information or beliefs in the light of new evidence. A belief revision system detects a conflict of belief when new belief contradicts the existing ones in a knowledge base. It resolves the contradiction between two beliefs to maintain the consistency of beliefs. It "*maintains*

---

<sup>5</sup> Also referred to as belief revision system in our discussion.

well - - -  
the truth in the system" as described in (Stanojevic & Vranes, 1994).

In belief revision systems, each piece of information involved is regarded as a belief. In educational systems, knowledge (that belongs to the student and the system) is the most central information. But, how do we relate knowledge and belief? Knowledge can be defined as *justified, true belief*. But in learner modelling, we deal not only with true beliefs but also incorrect (untrue) beliefs. For that reason, Self suggests that:

*"If we take knowledge as "justified, true belief", then it is contradictory to regard the learner model as describing the learner's knowledge when it includes descriptions of misconceptions. It would be more sound to regard the learner model as describing a learner's beliefs, not knowledge. ... The "education as growth" <sup>6</sup> model may then become partly describable in terms of a process of 'belief growth' or 'belief revision'."*

(Self, 1990), p. 6

This has been emphasised again when (Self, 1991) defines the terms knowledge and belief. Knowledge is really the intended, eventual outcome of learning. During the process of learning and at any intermediate stage, learners possess beliefs. And these beliefs may be untrue, inconsistent or incomplete. In other words, a belief is what the learner possesses during his learning process while knowledge is what the learner ought to gain as a result of his learning process.

As discussed in previous sections, inconsistency or contradiction of beliefs is a critical issue in learner modelling. Contradiction of beliefs in AI-ED systems may arise in the following situations:

1. the learner (either aware or unaware) has contradictions or conflicting beliefs in his reasoning process;
  2. the learner changes his mind or does not have consistent understanding during the learning process;
- not contradictory, per se

---

<sup>6</sup> "Education as growth" model is favoured by many major scholars such as Dewey, Paiget, Rogers for today's educational philosophy, compared to "education as transmission" which dominated in the previous three centuries (Self, 1990)

3. the learner has entrenched misconceptions that tend to resurface from time to time, which causes inconsistent performance;
4. the system's observation of the learner's performance is contradicted due to the learner's inconsistent performance.

As in many other ITSs, the premise in TRAPS' instructional interaction is heavily determined on the basis of the learner's interaction (or input). Hence, the learner model is a major component of the system. We focus our learner modelling in the work of handling contradictions. TRAPS uses a belief revision system in its learner modelling to handle the occurrence of conflicting information when the learner commits a reversal error when translating relational algebra word problems and when TRAPS makes an overall observation of the learner's performance through out his on-line session.<sup>7</sup> TRAPS applies the belief revision system for all of the four situations mentioned above that may arise.

In TRAPS, we deal with learners' beliefs and TRAPS' beliefs. The learner's beliefs, which are his answers to TRAPS' questions, may be incorrect and inconsistent. TRAPS' beliefs which includes **beliefs about the domain knowledge**, **beliefs based on implication of the learner's beliefs (answers)**, and **beliefs about the learner's performance**. TRAPS' beliefs about the learner's performance may be incorrect and inconsistent too. But, beliefs about the domain knowledge and beliefs based on implication of the learner's answers are assumed to be correct, as an expert's (tutor's) belief which can be regarded as TRAPS' knowledge. However, in order not to complicate the term knowledge and beliefs in the discussion of belief revision, we refer to TRAPS' correct knowledge as TRAPS' beliefs also.

## 2.6 Conclusion

We have discussed learner modelling as a component that aims to individualise a system's interaction with its users (learners). Issues related to learner modelling have been mentioned with particular focus on inconsistency of information that may occur

<sup>7</sup> This will be discussed further in chapter 5.

in the process. The function of belief revision and its role in assisting learner modelling has been described.

The following chapter describes relational algebra word problems, a domain where contradiction of information may arise in the learners' reasoning process and where there is a tendency to occurrence of inconsistency performance. Such a domain is used as an example to illustrate where and how a belief revision system can be useful for learner modelling.

## Chapter 3

# Algebra word problems

This chapter describes the background on the domain that we are dealing with, algebra word problems. We start off with the characteristics of algebra word problems and their norms and structure. Discussion then proceeds with relational type of algebra word problems and errors related to them. Some cognitive phases and common strategies applied by problem solvers are described. This is then followed by a description of the problem solvers' performance that demonstrates their conceptual understanding. The chapter ends with some existing work that deals with word problems.

### 3.1 What is an algebra word problem?

Some characteristics of algebra word problem are given as follows. They consists of one or more sentences having some known and unknown values with an underlying source formula.

Example:

*If a truck travels 10 hours at 30 miles per hour, how far will it go?* This has an underlying source formula of  $Distance = Rate \times Time$ .

They can also be presented in story lines, which consist of characters, actions and/or objects. This sometimes involves the mixture of relevant and irrelevant facts.

Example:

*Lemonade costs 95 cents for one 56 ounce bottle. At the school fair, Bob sold cups holding 8 ounces for 20 cents each. How much money did the school make on each*

bottle?

The domain of algebra word problems is by nature an open domain requiring common sense reasoning and the inference of unstated information. As in the case of the algebra word problems shown below, the problem solver has to use his common sense knowledge of the words *downstream* and *upstream* in order to solve the problem.

Example:

*A river steamer travels 36 miles downstream in the same time that it travels 24 miles upstream. The steamer's engine drives in still water at a rate of 12 miles/hour more than the rate of the current. Find the rate of the current.*

An outstanding aspect of algebra word problems is that solving them has always been a stumbling block for many students. The National survey of Mathematics Problem Solving in the United States has shown that 29% of 17 years old students could not solve the lemonade algebra word problems above (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). Children also perform 10% to 30% worse in word problems<sup>1</sup> than comparable problems presented in numeric format (Carpenter et al., 1980). Such a criteria has also been highlighted by Weaver III, *algebraic*

*"Word or story algebra problems are hard. Even students who are good at mathematics often hate them, and neither teachers nor text books know how to teach them: typically they give such well-meaning advice as to "read and reread the problem until what is stated is clear" (Fuller, 1977), p.115, and otherwise are satisfied to provide students with an opportunity to practice."*

(Weaver III & Kintsch, 1988), p. 2

### 3.2 Norms and structure of algebra word problems

Mayer's (Mayer, 1981) compilation and analysis of a thousand and ninety seven algebra word problems from ten standard algebra text books classified word problems into two categories.

---

<sup>1</sup> The experiment was on arithmetic word problems. Algebra word problems are more complex than they require multiple steps in the derivation of an equation as well as its solution, if compared to that of arithmetic word problems (Weaver III & Kintsch, 1988).

At the higher level, they can be classified by **families**. A family is a group of problems that share a certain basic underlying source formula. Among the twenty five families are *Time rate*, *Geometry*, *Physics*, and *Number story*. The underlying formula could be  $distance = rate \times time$  for the *Time rate* family, simple physics laws such as Ohm's Law,  $R = V/I$  for physics word problems. However, the *Number story* problems are not based on any specific source formula. Some of these families consist of several categories themselves. As in Figure 3.1, two categories of the *Time-Rate* family are shown.

At a detail level, an algebra word problems can be decomposed into a list of propositions. There are three major types of propositions:

**assignment propositions** - involve giving a single numerical value for one variable.

Examples:

- a) The price of the land is 25 dollars per square feet.
- b) The time to fill one pipe is 5 hours.

**relation propositions** - involve giving a single numerical relationship between two variables.

Examples:

- a) The boat travels 12 miles/hour more than the rate of the current.
- b) Migraine among adults is twice as common as asthma.

**question propositions** - find a single numerical value corresponding to a given variable of the word problem.

Examples:

- a) How much time does it take to fill in half the tank?
- b) What is the number of adults with migraine?

A list of propositions form a **template**. Figure 3.2 shows a *Time rate* family type of algebra word problem with its derived template.

Time rate Family (distance = rate x time)  
=====

Motion problem  
-----

A train leaves a station and travels east at 72 km/h.  
Three hours later a second train leaves on a parallel  
track and travels east at 120 km/h.  
How long will it take to overtake the first train?

Current problem  
-----

A river steamer travels 36 miles downstream in the  
same time that it travels 24 miles upstream. The  
steamer's engine drives in still water at a rate of  
12 miles/hour more than the rate of the current.  
Find the rate of the current.

---

Figure 3.1: The *Time-Rate* family word problems.

Simple DRT problem  
-----

Bill & John drives every Monday from Boston to Cleveland,  
distance of 600 miles. It takes John 10 hours to drive  
from Boston to Cleveland. If it takes 2 hours more for  
Bill to do so, what is Bill's rate of driving?

The template :

object1	=	Bill	-> assignment proposition
object2	=	John	-> assignment proposition
distance	=	600	-> assignment proposition
time-object2	=	10	-> assignment proposition
time-object1	=	2 + 10 = 12	-> relation proposition
rate-object1	=	Find	-> question proposition

---

Figure 3.2: Decomposition of the *Distance-Rate* word problem into a template.



### 3.3 Relational algebra word problems

There are word problems that use relational propositions in their story or non-story lines. These are referred to as relational algebra word problems. Relational algebra word problems involves a single numerical relationship between two variables. Any category of word problem family can be represented in a relational form of statement. In our research, we deal with the *Number story* type of word problems with relational statements in them. The *Number story* category of word problems is defined as “One or more sentences presented, containing one or more unknowns with no story line. The task is to solve for an unknown.” (Mayer, 1981). They are not based on any specific source formula.

However, we are interested in the relational *Number story* with some story lines, with no specific source formula, where the task is to formulate an equation from the statement rather than solving for an unknown value.

An example of algebra word problem we deal with:

*“There are twice as many adults suffering from migraine as adults suffering from asthma”*. Translate this statement into an algebraic equation using the variable M for the number of adults suffering migraine and A for the number of adults with asthma.

Concepts and common misconceptions related to such a relational word problem will be discussed in detail in section 3.6.

#### 3.3.1 Empirical studies

Many empirical experiments and studies have been carried out by mathematician and cognitive scientists to find out the reasons of algebra word problems being a stumbling block for many students. The following are some of their findings.

(Greeno, 1980) explicitly showed that primary school children had more difficulty in translating sentences that involved relational information. For example, in the problem *“Joe has 3 marbles. Tom has 5 more marbles than Joe. How many marbles does Tom have?”*, the students recall the same problem as *“Joe has 3 marbles. Tom has 5 marbles. How many marbles does Tom have?”*. However the children were quite

proficient at repeating problems in which each sentence dealt with one variable i.e without a relation between two variables.

(Loftus & Suppes, 1972) located “structural variables” that affected the difficulty of story problems for sixth graders. As examples, the difficulty of a problem was increased if it was a different type from the previous one, if it required many arithmetic operations, and if the syntactic structure of the sentences was complex. They also found that the hardest problem in their set was one that contained a relational proposition eg. “*Mary is twice as old as Betty was 2 years ago. Mary is 40 years old. How old is Betty?*”

Relational information is more difficult to mentally represent than other kinds of relevant information in a word problem (Mayer, 1982). Mayer’s analysis had also proposed the **Propositional Structure Hypothesis (PSH)** which states that **assignments are psychologically more basic than relations.** ~ in what sense?

(Clement, Lochhead, & Soloway, 1979) have shown that difficulties in translating relation propositions occurs not only among school children but also among college students. They asked the students to solve the problem:

*“There are 6 times as many students as professors at the university. Write an equation using  $S$  as the number of students and  $P$  the number of professors”.*

Here Clement et al. found that the common mistake made by the students is due to their confusion about the semantics of the algebraic equation. This then leads the students to adopting the incorrect strategy while translating the sentence. On the other hand, an interesting discovery in this experiment is that when students were asked to translate the same relational statement by writing a computer program, the error rate fell dramatically (Clement, Lochhead, & Soloway, 1980). Such a finding suggests that people have difficulty in interpreting what a relational proposition means when they must use a static format such as simple sentences.

unlike a computer program?

### 3.3.2 A further survey

A small survey on translating relational algebra problems was conducted by the author in December, 1993. Subjects in this study were adults with tertiary level of maths and

science background. The materials of relational algebra problems consist of questions that required the subjects to translate relational statements, data in a table format and picture format into algebra equations, and also to translate a simple linear equation into a statement in English. A set of questions for this survey is attached in Appendix B.

The aim of the survey was:

1. to find out whether relational problems (in various forms of data) are considered difficult among adults with such a background;
2. to observe adults' strategies in solving the problems given.

Two out of six subjects committed the *reversal error* by formulating the *reversed equation*. Those who <sup>did</sup> not took a long time in answering each of the questions and they double checked their equation by plugging numbers into the variables. The strategies<sup>2</sup> they apply are *word order matching*, *operative approach* and *substituting numbers*. All of them agree that relational algebra problems require some serious thinking before translating them.

As can be seen from the literature and the survey conducted, relational propositions or statements appear to be the cause of difficulties that many people face with word problems.

### 3.4 Errors of relational propositions

There are several errors committed by students when attempting to solve algebra word problems with relational propositions. Among the frequent errors they make in relational propositions are:

- **Conversion Error** (Mayer, 1982)

The relational proposition is changed to an assignment proposition.

Examples:

---

<sup>2</sup> Discussion of these strategies is in section 3.6.

Given problem:

The steamer's engine drives in still water at the rate of 12 mph more than the rate of the current.

Student recall:

In smooth water the engine causes it to move at 12 mph.

- **Reversal error** (Clement et al., 1979)

The learner produces an equation which implies the reverse of an algebra interpretation of a correct equation.

*not well expressed*

Examples:

*There are six times as many students as professors. Translate this statement into an algebra equation, using  $P$  (for number of professors) and  $S$  (for number of students) as variables in the equation.*

Typical incorrect answer:  $6S = P$

Correct answer :  $S = 6P$

Experiments with the “*student-professor problem*” had been carried out among engineering undergraduate students. Results were that 37% of them answered incorrectly and 68% of the incorrect answers were the reverse equation i.e  $6S = P$  (Clement, 1982).

Based on the “*student-professor problem*” and the analysis of empirical experiments done by (Clement, 1982), (Rosnick, 1981) and that of the author, the majority of the experimental subjects believe qualitatively that there are more students than professors. However, the formulated equation which is a reversed equation implies the opposite *qualitative value*. We defined *qualitative value* as a conceptual view that describes which of the two groups involved in the given problem is larger<sup>3</sup> in size or quantity than the other. In this particular example, *there are more professors than students*.

---

<sup>3</sup> This attribute may differ (eg. smaller) depends on the context of the problem statement given

### 3.4.1 Emphasis on the reversal error

Of the two frequent errors of relational propositions, the reversal error is found to be a more serious problem among the majority of students and adults.

Below are some of the issues that support the need to concentrate on the reversal error.

- The reversal error stems from misconceptions concerning the structure and interpretation of algebraic statements. Such misconceptions are usually deeply seated and not easily dislodged by the student (Lochhead & Mestre, 1988). The misconceptions tend to resurface consciously or unconsciously within the students' reasoning process.
- The misconceptions related to the reversal error are not easily rectified in a moderate length of tutoring session (Rosnick & Clement, 1981). The misconceptions emerge again when problem solvers are provoked further on the equation and in methods they apply in translating word problems. The misconceptions are persistent (Clement, Lochhead, & Monk, 1981) or entrenched in many of the problem solvers' reasoning processes.
- These algebraic misconceptions are not only typical errors among college students but also widespread across academic background, age and nationality as indicated below:
  - mathematics, science, engineering majors (Lochhead & Mestre, 1988);
  - non-science majors (Clement et al., 1981);
  - 9th graders and higher level (Mestre & Gerace, 1986);
  - Hispanic students (Mestre, Gerace, & Lochhead, 1982);
  - Fiji, Israel, Japan (Lochhead, Eylon, Ikeda, & Kishor, 1985) and (Mestre & Lochhead, 1983).
- Some disturbing matters that have been concluded by among others (Clement et al., 1981) and (Rosnick & Clement, 1981), in their research:
  - Engineering and Science students have missed the mathematically essential notions of equation and variable;

- Science oriented students are confused at the interface between algebraic symbols and their meaning;
- The above two problems exist in an even larger proportion of non-science students;
- Writing correct equations does not necessarily always imply the students' correct understanding. This suggests that some large numbers of students may be slipping through their education with good grades but little learning.

The above disturbing issues suggest that the instruction in algebra generally fails to provide students with an adequate opportunity to learn how to interpret mathematical symbol strings. This places them at a severe disadvantage in the interface between mathematical symbols and description of real word problem. This also hinders the students when it comes to learning the symbol manipulation rules of algebra (Rosnick & Clement, 1981).

### 3.5 Solving algebra word problem

Two cognitive phase<sup>S</sup> involved in solving word problems as described in (Mayer, 1982) and (Paige & Simon, 1966):

**Translation** - understanding the problem, as manifested in translating the words of the problem into an internal representation in memory.

i.e. Text  $\rightarrow$  Memory representation  $\rightarrow$  Equation

**Solution** - applying the legal rules of algebra and arithmetic to the internal representation to deduce the answer.

i.e. Solving the equation to get one or more numerical value<sup>S</sup>

Between the two process mentioned, Simon's research suggests that the major difficulty lies in the translation phase although most instruction focuses on the solution phase (Simon, 1980). Our work here however, focuses on the translation phase.

A number of common concepts and misconceptions that many students encounter while translating the *student-professor* type of relational word problem to an equation have



been identified (Rosnick & Clement, 1981); and (Clement, 1982). Such misconceptions then lead the students to adopting the incorrect strategy when translating a sentence. Some of the common incorrect strategies applied by the students are: the **word order matching** or **direct mapping** approach and the **static comparison** method. On the other hand, the common correct strategies which lead the student to formulate the correct equation (i.e correct translation) are: the **operative** approach and the **ratio** approach<sup>4</sup>. The description of these strategies is discussed further in the following section and summarised in Table 3.1.

### 3.6 The underlying cognitive process

Correct and incorrect conceptual understanding of students when translating relational algebra word problems have been found in empirical studies such as that by (Rosnick & Clement, 1981), (Clement, 1982) and (Lochhead & Mestre, 1988). These findings reveal some common underlying cognitive processes involved when people translate relational algebra word problems.

The following are the **common incorrect methods** of translation that lead many problem solvers to formulate a reversed equation from a relational algebra statement. Misconceptions that relate to such approaches are described too. We refer to the *student-professor* question as the example in the discussion below.

#### 1. Word order matching approach.

The problem solver performs a left-to-right keyword order matching. They mechanically match the order of symbols in words in the problem statement to the order in the equation. Another name for word order matching is **direct mapping** which describes the process of mapping directly the keywords in the statement to a string expression (an algebraic equation) which forms an equation.

Such an approach is a syntactic strategy. It is purely based on rules of arrangement of symbols in an expression that do not depend on the meaning of the expression. In other words, the word order matching strategy lacks a semantic

---

<sup>4</sup> This approach has been observed from the author's survey

process.

The misconception:

The order of the keywords in the problem statement maps directly into the order of symbols appearing in the equation.

Examples of problem solvers' statement that support this approach: <sup>5</sup>

⌚ "Well, the problem states it right off: ⌚6 times students'. So it will be 6 times  $S$  is equal to professors."

⌚ "The problem just says it that way."

⌚ "You just get it directly from the words."

**2. Static comparison approach.**

This is another step beyond the syntactic word order matching. Here, the problem solver uses a semantic approach dependent on the meaning of the problem. He makes a literal attempt to compare the relative sizes of the two groups in a static manner. A concept called the *figurative concept of equation* (Rosnick & Clement, 1981) where the coefficient is being associated with the ⌚bigger variable", and hence forms a reversed equation. As an example, in the *student-professor* word problem, the coefficient 6 is attached to  $S$ , since  $S$  is the larger group compared to  $P$ . Such an approach also demonstrates that the problem solver sees the letters  $S$  and  $P$  as the physical object of the group and not as an unspecified quantity. There is a confusion in the function of the letter as variables and as labels.

The misconceptions:

- (a) The variable is used as label for the object in the problem statement rather than standing for an unknown and unspecified quantity.
- (b) The equal sign means a correspondence or association between unequal groups rather than symbolizing an equivalence (in amount) of a group in the left side and another group on the right side of the equation.

---

<sup>5</sup> Based on i) the students' responses in (Rosnick & Clement, 1981)'s interview with their experiment's subjects, ii) from the author's observation made when conducting Study 2 formative evaluation, as described in section 7.4.4



- (c) The coefficient acts as an adjective or prefix rather than as an operator or a number.

Examples of problem solvers' statement that support this approach: <sup>6</sup>

*"S stands for students, and P stands for professors".*

*"Put the number next to the variable to make it a larger group".*

*" $6S = P$  means six students for every professor".*

In contrast, problem solvers who have correct conceptual understanding that enable them to formulate correct equations from relational word problems tend to apply a correct method. The following are some **common correct strategies**:

### 1. Operative approach

The problem solver multiplies or divides the number in one group by a constant in order to make it the same amount as the number in another group. Here, the problem solver views the equation as representing an operation on a variable quantity to produce a number equal to another unspecified quantity.

Such a strategy leads to the correct algebraic equation and demonstrates that the problem solver has the correct conceptual understanding in translating the word problems.

### 2. Ratio approach

The problem solver forms the ratio on both groups involved in the problem statement, then a cross-multiply operation between the variables and the ratio values is performed.

This approach leads the problem solver to formulating the correct equation. Whether the approach suggests that the problem solver has the correct conceptual understanding, is unclear. Most problem solvers who apply this approach, counter check the equation formulated by plugging in numbers and match that to their qualitative understanding.

---

<sup>6</sup> Resources are the same as from examples for word order matching.

*Step 200?*

### 3. Substituting followed by operative

In this method, the problem solver substitutes numbers into a trial equation in order to test it, then explains the meaning of the equation in terms of operative patterns. As pointed out by (Rosnick & Clement, 1981), this strategy helps the problem solver to produce the correct equation but however, it does not imply whether he understands his solution in the equation he formulates.

Table 3.1 summarises the strategies that have been described in this section.

Strategies	Description
1. Word order matching	Maps directly from the order of the keywords in the problem statement. This is done syntactically without understanding the meaning of the equation formulated.
2. Static comparison	Places multiplier next to the letter associated with the larger group. Lack of understanding on the functions of the variable, the coefficient and the equal sign is the underlying reason for this.
3. Operative	Views the equation as representing an operation on a variable quantity to produce a number equal to another unspecified quantity.
4. Ratio	Forms the ratio on both groups involved in the problem statement. Then a cross-multiply operation between the variables and the ratio values is performed.
5. Substituting numbers	Substitute numbers into trial formulated equations in order to test the equation.

Table 3.1: Common approaches that students apply in translating and formulating relational algebra word problems.

The underlying cognitive process in these approaches can also be described in terms of misconceptions versus missing concepts. In this context, we define *misconceptions* as the incorrect concepts and reasoning that the learner has, while *missing conceptions* as the correct concepts and reasoning that he lacks off. A summary of misconceptions

Missing conceptions	Misconceptions
1. The order of the key words in the statement maps directly into the order of symbols in the formulated equation.  2. The variable is the label for the object involved.  3. The equal sign symbolises a correspondence or association between unequal groups.  4. The coefficient acts as an adjective to be attached next to the variable.	1. Mapping directly the key words is only a syntactic process. It doesn't 'express' the meaning of the equation.  2. The variable is an unspecified number representing the quantity of the object in the problem statement.  3. The equal sign symbolises an equivalence.  4. The coefficient is a number operating on the variable to balance the left and right side of the equation.

Table 3.2: Problem solvers' misconceptions and missing concepts related to the reversal error.

and missing conceptions related to the reversal error can be represented as in Table 3.2. We group both misconceptions and missing concepts and refer to them as the **contradictory schemes** of learners who commit the reversal error.

### 3.7 Diagnosing the problem solvers' conceptual understanding

Having the correct or incorrect conceptual understanding plays an important role for the problem solver in translating word problems. We describe some of the common behaviour that demonstrates whether one carries correct or incorrect concepts in relation to solving this type of problem.

#### 3.7.1 Identifying incorrect conceptual understanding

(Rosnick & Clement, 1981) study of students committing reversal errors in relational algebra word problems has produced a list of observations on the students' behaviour when translating algebra word problems. Such observations are confirmed by a survey

conducted by the author as described in section 3.3.2. The problem solvers' behaviours which demonstrate their lack of conceptual understanding are illustrated as follows:

1. They remained incapable of writing the correct equation throughout the session.
2. After correcting the reversal mistake, at a later time, they:
  - revert back to the reversed equation.
  - accept the correct equation but switch the meaning of the original problem.
  - identify the correct equation as being "not making sense".
  - acknowledge that the correct equation "works" but state that they don't know why it works.
3. They read the correct equation erroneously. e.g. "One student for every professor".
4. After making a minor arithmetic mistake while checking the correct equation they immediately doubt and discard the correct solution before rechecking the arithmetic. This suggests that their belief of the correct equation is extremely tenuous.
5. They are unable to replace the letter with correct values. This demonstrate a clear misunderstanding of the use and meaning of letters in equations.
6. After apparently learning how to solve the more elementary problems, they either:
  - make no attempt to apply their learning to a more difficult problem, or
  - attempt to apply their learning but perform it erroneously.
7. They are unsure of which of the equations (the correct and reversed ones) is the right equation. They plug in numbers to check if the equation justifies their qualitative understanding. The qualitative understanding is the understanding of which of the two groups is bigger in terms of amount. The one that matches their qualitative understanding will be chosen as the correct equation.

### 3.7.2 Judging correct conceptual understanding

On the other hand, a criteria<sup>Be</sup> which suggests that a problem solver has the correct conceptual understanding helps him avoid committing the reversal error. The problem solver indicates that he understands that variables stands for numbers rather than individual objects and that the larger coefficient is associated with the variable that represents the smaller group in order to equalize both sides of the equation.

## 3.8 Assisting the problem solvers

As has been suggested in section 3.4.1, some people in their school days get through their mathematics courses but with perhaps little learning. Hence, there is a need to overcome such problem with our particular concern in translating relational algebra problems. Some suggestions have been made by (Rosnick & Clement, 1981) to assist problem solvers learn to **translate relational algebra word problems with understanding**:

1. Fundamental concepts of variable and equation should be treated as important in secondary schools and colleges.
2. Encourage problem solvers to view equations in an operative way - the equation represents active operations on variables that create an equality. This contrasts with the view of an equation as a static statement, where the larger coefficient is associated simplistically and incorrectly with the larger variable.
3. It is essential that problem solvers be able to view variables as some unknown representing numbers.
4. Attention must be paid to conceptual development in mathematics education.

## 3.9 Existing research in word problems

As has been discussed in several sections in this chapter, **“word problems are found by many students difficult to interpret<sup>e</sup>, translate and solve”**. In relation

to this, interest have been shown by several researchers from various backgrounds eg. educationists, psychologists and computer scientists implementing computer-based systems in the domain of word problems for arithmetic, algebra and mechanics.

Example of earlier systems are that of (Bobrow, 1974)'s STUDENT, and (Bundy, Byrd, Luger, Mellish, Milne, & Palmer, 1979)'s MECHO. Bobrow's STUDENT system works by translating English sentences directly into equations. He uses STUDENT to discover how one could build a program which could communicate with people in natural language within a small problem domain. Bobrow applies tagging to each word and symbol to indicate the grammatical function these words perform. The tags are checked against a dictionary look-up table. The technology and method available during that time limits STUDENT to only deal with simple problems.

Bundy et al's MECHO system solves mechanics problems - from a statement in English or from predicate calculus assertions describing a physical situation. MECHO is used to test several AI techniques for controlling the search involved in parsing English and performing inferences. The mechanics problems MECHO deals with are the pulley problems, motion on smooth complex paths and motion under constant acceleration.

Some other recent work is that of (Nathan, 1991); (Singley, 1989); (Derry & Hawkes, 1993); and (LeBlanc, 1993). Their work is concerned less with the algebraic manipulation involved in the problem. They stress the cognitive process by which the problem solvers transform their mental model of the problem into an algebraic expression.

(Nathan, 1991) developed a system called ANIMATE (An Interactive system for Mapping Animation to Text), an interactive computer animation-based tutor. The tutor's feedback is unobtrusive and interpretive: unexpected behaviour in the equation-driven animation of a situation highlights equation errors which the student resolves through iterative debugging.

(LeBlanc, 1993) worked on a system for cognitive model of mathematical reading. He concentrated in the domain of arithmetic word problems. There are two major components in his system — EDUCE and SELAH. EDUCE is an expectation-based conceptual analyser which reads or parses each sentence of the problem in a word-by-word manner. SELAH does the text integration by performing text integration across



System	Work Concentration
STUDENT (Bobrow, 1974) MECHO (Bundy et al., 1979) AWPT (Singley, 1989) TAPS (Hawkes et al., 1993) ANIMATE (Nathan, 1993) EDUCE and SELAH (LeBlanc, 1993)	Natural language processing (NLP) NLP and search control Psychological investigation tool Cognitive tools Visual interface Cognitive model

Table 3.3: Related works in the domain of word problems.

sentences to establish the connections between sentences and ultimately the relationship between sets. LeBlanc’s aim in his work is to provide a computer environment that is flexible enough to give teachers the control necessary for classroom use.

(Singley, 1989) works on the Algebra Word Problem Tutoring (AWPT) system. His tutoring system is based on the ACT\* (Adaptive Control of Thought Star) theory of learning (Anderson, 1983). It provides corrective help and feedback on errors as student solve an algebra word problem.

(Derry & Hawkes, 1993) TAPS (Translating Algebra Problems System), TAPS project attempts to develop students’ metacognitive intelligence through methods that combine apprenticeship philosophy with graphics-based computer technology. It is a computer based cognitive tool designed to assist students, of varying grades and ability levels, strengthen the type of planning and self-monitoring knowledge. (Hawkes, Derry, Diefenbach, & Kegelman, 1993) believe that planning and self-monitoring is an important aspect of complex problem solving.

The various systems mentioned above concentrate on different issues that relate to word problems. Even though their focuses are different, the common goal shared in these systems is to provide assistance either to teachers or students in a domain where many people find it difficult — word problems which involve equations (for algebra, arithmetic or mechanics).

### 3.10 Conclusion

In this chapter, we have discussed in detail the characteristics of algebra word problems and why algebra word problems are difficult for many people. Particular focus on relational algebra word problems has been made in this chapter. Correct and incorrect conceptual understanding related to translating relational algebra word problems has been described.

Having discussed in detail the above issues, relational algebra word problem has been the choice of domain in this research for the following reasons: ✓

- Daily problems are often described in statements or word problems even though word problems are found to be difficult by many people; Algebra VC
- Relational statements in word problems are among the common difficulties faced by many people;
- Misconceptions related to the reversal error are entrenched misconceptions;
- Misconceptions related to the reversal error leads to contradiction of beliefs which suits the application of TRAPS' learner modelling technique<sup>7</sup>.

This domain is found to be suitable for us to illustrate the application of belief revision in learner modelling. The following chapter discusses the belief revision system which is seen to be an appropriate AI technique to model contradictions and entrenched misconceptions, two major characteristics of the domain, translating relational algebra word problems into equations.

---

<sup>7</sup> This will be discussed in chapter 5.



## Chapter 4

# The Belief Revision System

In this chapter discussion on the core of our learner modelling, i.e the belief revision system is made. We introduce the two belief revision systems involved in our work — *Belief Revision System (BRS)* and *Belief Revision In Learner Modelling (BRILM)*. The former has been the basis for the latter system. Description of *BRS* in terms of its underlying theory, formalism, implementation and features is presented. These are then proceeded by describing the modification made to produce *BRILM*. *BRILM*'s capabilities and limitations are also discussed.

### 4.1 The two belief revision systems

As described in chapter 2, a belief revision system deals with contradiction of information or beliefs in the light of new evidence. A contradiction is defined as the occurrence of a belief,  $b$  and its negation,  $\neg b$  at the same time. Having detect an occurrence of contradiction of beliefs, the belief revision system is responsible for resolving it by retaining one of the conflicting beliefs (either  $b$  or  $\neg b$ ) and its justifications.

Berendt's system (Berendt, 1992) is a belief revision system that combines both the foundational and coherence approaches<sup>1</sup> in detecting and revising conflicting beliefs. A preference ordering, called *epistemic entrenchment*<sup>2</sup> is used to resolve the conflict. We will refer to her system as the **Belief Revision System** or *BRS*. TRAPS incorporates a modified version of BRS for its (TRAPS) learner modelling. The modified version is

---

<sup>1</sup> This is explained in the following section.

<sup>2</sup> This is defined in section 4.3.2.

called the **B**elief **R**evision **I**n **L**earner **M**odelling or *BRILM*.

Three main methodological issues in a computational setting need to be considered when handling belief revision as described by (Gärdenfors, 1992):

1. Representation of the beliefs in the database.
2. The relationship between fundamental beliefs and the derived beliefs.
3. How are the choices made concerning which of the contradictory information is to be retracted? Logic alone seems not to be sufficient for making such a decision. One or more extralogical factors are needed. What are the possible extralogical factors and what technique are useful here?

Our discussion on both belief revision systems, BRS and BRILM, will touch on each of the above matters with special focus on the third issue. This chapter in general will discuss the underlying theory and mechanism of Berendt's BRS<sup>3</sup> and the necessary changes made to produce our BRILM.

## 4.2 Foundational and coherence approaches

When discussing the theory of belief revision, there are two major well known approaches — the *foundational* and *coherence* approaches. In the foundational approach, a proposition (a belief in this context) is believed if and only if one or more justifications can be exhibited for the belief. The coherence approach views information in a more global structure where a proposition is believed if it is coherent with other existing beliefs. Justifications are not important in the coherence approach. Coherence of beliefs can be considered from several points of view — logical consequences (Berendt, 1992), “derivability of core beliefs” (Galliers, 1990), or supplemented by other relations of implication and explanation (Harman, 1986).

Two types of belief are distinguished in a foundational system. A *foundational belief* is a self-evident belief which does not require any justifications. It serves as the last point in a justification of any beliefs related to it. The other type of belief in the

---

<sup>3</sup> For a full description, see (Berendt, 1992).

foundational approach is a *derived belief*, which is inferred from one or more other beliefs (its justifications). In a coherence system, the distinction between foundational and derived beliefs is not made. The important concept in a coherence theory is that the beliefs are related to each other.

In terms of belief changes, in a foundational theory all beliefs that are no longer justified are given up and all old and new beliefs that become justified are added. Belief change in a coherence system generates a new belief state that is consistent by the principles of *conservatism* or *minimal change* and *maximal coherence*. According to Gärdenfors conservatism is similar to minimal change i.e one should continue to believe as many as possible of the old beliefs, when changing beliefs in the light of new evidence. This is then referred to as *informational economy* (Gärdenfors, 1988). Maximal coherence refers to having the largest number of beliefs that cohere. Maximal coherence is further defined by Gärdenfors as:

*“pragmatic consideration - inconsistent sets of belief are of no help when seeking guidance for how to act, and, in order to use one’s knowledge effectively, one must be able to draw the consequences of the information one has on a topic”*

(Gärdenfors, 1988), p. 22

Classic examples for the foundational system are *Reason Maintenance Systems (RMS)*<sup>4</sup> (Doyle, 1979); (Doyle, 1992) and for the coherence approach is the *AGM*<sup>5</sup> postulates (Alchourrón, Gärdenfors, & Makinson, 1985), hence the name.

Despite the differences, both foundational and coherence approaches provide valuable contributions. Reason maintenance focuses on the reasons relating individual beliefs while the AGM approach focuses on the ordering of beliefs according to the principle of epistemic entrenchment. *not inherent in AGM.*

Doyle in his analysis on coherence and foundational methodologies (Doyle, 1992) concludes that AGM and RMS, representing the two approaches respectively, can be seen

<sup>4</sup> Also called as *Truth Maintenance System (TMS)*. However, the term TMS is a misnomer that could mislead its actual function and be better referred to as Reason Maintenance System (RMS) (Doyle, 1983). We will use RMS in our discussion.

<sup>5</sup> Taken from the original work by Alchourrón, Gärdenfors and Makinson, hence the name.

as:

- Both seek to recognise logical and inferential relations among beliefs, differing at most in how these relations are presented. Both must abandon most requirements of logical consistency and closure to be useful in practical implementations.
- Both make minimal changes of belief, differing at most in the set of possible alternatives entering into the minimization.
- Both allow flexibility in choosing whether to reflect reasons and other inferential relations in epistemic states, differing only in whether representations of reasons determine entrenchment orderings or representations of entrenchment orderings determine reasons. Both make no stipulations about what reasons or entrenchment ordering should be represented, other than to assume this information may change with each step of reasoning.
- Both allow one to ground beliefs only to the extent required by one's needs for explanation and updates, and to change the identification of basic beliefs along with the current purposes of reasoning.

Having said that both of the foundational and coherence theories share some similar fundamentals, Doyle concludes that if given computational considerations, the foundational approach is *"the most practical means of mechanizing coherence approaches"* (Doyle, 1992), p.48.

BRS' combination of the foundational and coherence approaches is realised by its application of RMS along with a formalism which satisfies the AGM postulates. Such a combination is intended to take the advantages of both foundational and coherence approaches. <sup>6</sup>

✓ not all of them.

As in the foundational approach, it provides the features of:

- Clearer style of reasoning - All beliefs are either **justified** (for derived beliefs) or **assumptions** (for foundation beliefs). This reasoning approach is a systematic way of providing explanations and is compatible with human reasoning.

---

<sup>6</sup> There are other work such as (Galliers, 1992) and (Cawsey, Galliers, Reece, & Jones, 1992) which integrate features of both approaches.

- Minimal data requirement - A preference ordering (epistemic entrenchment value) needs only to be defined over assumptions which form a subset of all beliefs. A derived belief relies on one or more argument that justifies it. Evaluation of the entrenchment of this belief depends on the arguments for it and the entrenchment of the beliefs in the arguments. Applying this recursively means that ultimately, only degrees of entrenchment of assumptions are needed when evaluating the derived belief's entrenchment. Unlike in a coherence approach where every belief must explicitly be assigned a degree of entrenchment.

Some positive characteristics that the coherence approach provides for BRS:

- Clear semantics is able to be provided due to the coherence approach having the advantage of applying a declarative logic. ~ but not to the revision itself.
- 87. • Allows more than one choice when revising beliefs. Either the belief  $b$  or its negation,  $\neg b$  can be believed when resolving to reach a consistent belief set.

suggests use deterministic

## 4.3 BRS' formalism

### 4.3.1 Basic principles

The belief state<sup>7</sup> of an agent is modelled by a set of statements. This set of statements is referred to as the *belief set*. The term statement in our discussion here refers to the belief in the belief set. A belief  $b$  can be in three belief states:

**Accepted** -  $b$  is fully believed.  $b$  is a member of the belief set.

**Indeterminate** -  $b$  is disbelieved.  $b$  is not a member of the belief set.

**Rejected** - the negation of  $b$  is accepted.  $\neg b$  is a member of the belief set.

Berendt finds the belief state, **rejected**, rather redundant since it can be expressed in terms of  $\neg b$  being accepted. In BRS, the belief state involved are accepted and indeterminate.

<sup>7</sup> A representation of an agent's knowledge and belief at a certain point of time. Also known as epistemic state in (Gärdenfors, 1988).



The belief set in BRS should satisfy two rationality criteria:

1. The set of accepted beliefs should be *consistent* i.e a belief  $b$  and  $\neg b$  should not exist together in the set.
2. Logical <sup>consequence</sup> consistent of what is accepted should be accepted too. ?

Two types of belief changes are in used in BRS:

**Expansion** - an indeterminate belief becomes accepted.

**Contraction** - an accepted belief becomes indeterminate.

Another kind of belief change applied by Gärdenfors is **revision** (Gärdenfors, 1988). Berendt interprets revision as a contraction followed by an expansion (Berendt, 1992), p.25. An example is changing from believing  $b$  to  $\neg b$  that takes disbelieving  $b$  first and followed by believing  $\neg b$ . Hence, the belief change in BRS is sufficient to be presented in terms of expansion and contraction only.

It should be noted that BRS is predominantly a foundational system rather than a coherence one.<sup>8</sup> Being a foundational system, beliefs are closed under logical consequences. Hence, the belief sets can only be either expanded or contracted by assumptions.

It is assumed that a belief system changes only when there is new information. If the new information is consistent with the existing believed information, expansion will occur, i.e the new information will be added into the belief set. If it is inconsistent with the existing believed information, changes have to take place in order to reach a new consistent set of beliefs. If the new information is too weak to challenge existing beliefs, it will then be ignored. On the other hand, if it is strong enough in the competition, other weaker beliefs and the justifications will be dropped from the belief set, i.e a contraction takes place. The strength of beliefs is measured by the ordering of epistemic entrenchment over the assumptions. In BRS' algorithm for belief changes, a contraction will only take place when the belief set becomes inconsistent.

---

<sup>8</sup> The choice for this has been justified further in (Berendt, 1992), p.40.

well -

In BRS, the belief states, epistemic inputs<sup>9</sup> and belief changes are all modelled in a logic-based formalism. The belief states are modelled as sets of believed statements, where the statements are from a logical language. The agent is assumed to have a set of assumptions, i.e. statements believed without any justification. In other words, assumptions are self-evident beliefs. Certain inference rules are assumed to determine which other beliefs are justified by the assumptions. These inference rules determine the notion of **logical consequence** and **consistency**.

### 4.3.2 Epistemic entrenchment

*Epistemic entrenchment* as the decision criteria when resolving a conflict of beliefs is one of BRS' conspicuous features. Epistemic entrenchment is defined as:

= total ?

*"Epistemic entrenchment is characterised by a complete preorder (a reflexive and transitive relation) over propositions which indicates which propositions are more valuable than others. This ordering influences revisions by the requirement that revisions retain more entrenched beliefs in preference to less entrenched ones. It may change over time or with the state of belief."*

(Doyle, 1992), p. 32

In a belief revision system with epistemic entrenchment, preference ordering over beliefs is employed when resolving a conflict. Each belief has an entrenchment value assigned. The higher the value, the more entrenched or valid it is to the system. The less tenacious belief of the two conflicting ones will be dropped, to maintain the consistency of beliefs in the system. The representation and formalism for epistemic entrenchment is described in section 4.6.2.

---

<sup>9</sup> Information provided through deliverance of experience or by another party (individual/machines) which may lead to changes of epistemic states.

## 4.4 AGM postulates in BRS

The AGM postulates manifest the notion of epistemic entrenchment introduced by Gärdenfors in (Gärdenfors, 1988). The underlying motivation of these postulates is that when someone changes his beliefs, he would want to retain as many as possible of his old beliefs, i.e making a minimal change. Unnecessary loss of information should also be avoided, i.e the criterion of informational economy.

It has been found that it is not easy to provide a precise quantitative definition of the loss of information.<sup>10</sup> For that reason, an approach to minimal change assumes that the statements in a belief set have different degrees of epistemic entrenchment. One should then give up the statement with the lowest degree of entrenchment.

Fundamentals of BRS correspond closely to AGM (Alchourrón et al., 1985) postulates. However there are some differences:

- BRS's logic emphasises the foundational approach while AGM emphasises the coherence approach.
- The foundational approach of BRS assigns degrees of epistemic entrenchment only to assumptions while AGM assigns them to all statements. BRS's approach here is able to limit the number of postulates that are applied and impede the proof of equivalence between the postulates for contractions and those for epistemic entrenchment. ?
- BRS's ordering of epistemic entrenchment is a strict ordering which differs from AGM's ordering. Details of Berendt's decision of using a strict ordering can be found in (Berendt, 1992), pp.65-70.

## 4.5 Reason Maintenance System in BRS

A reason maintenance system is a subsystem of a general system. It is not a reasoning system, but a support for such a system (BRS in this case) which does the reasoning itself.

---

<sup>10</sup> An example is the discussion on minimality in (Gärdenfors, 1988), pp.66-68.



A reason maintenance system traces the consequences of initial changes through records of inferences. It keeps track of the derivation of information to other information. Such a reconstruction of derivable information is the mechanism that a reasoning maintenance system helps revising the BRS' knowledge base states (from inconsistent to consistent).

The RMS consists of a network whose nodes represent beliefs and whose arcs represent inferences that lead from assumptions to derived beliefs, and from derived beliefs to further derived beliefs. The nodes represent the dependencies in the justification network as in Doyle's Reasoner Maintenance System (RMS) (Doyle, 1979). The arcs are stored in the data structure of justifications at the nodes.

The RMS in BRS has been simplified from the original Doyle's RMS by restricting justifications to Support List (SL) justifications. There are no Conditional Proof (CP) justifications.

In BRS, the reason maintenance (Berendt refers to it as *truth maintenance*) is the process of propagating changes in the support status of a node forward in the network. The process is triggered by either contraction or expansion. Contraction corresponds to a situation when an assumption node changes from *in* to *out*, while an expansion corresponds to a situation where an assumption node changes from *out* to *in*.

BRS's *dependency-directed backtracking* is the process of resolving a conflict which consists of:

- detecting a conflict when  $b$  and  $\neg b$  are both believed.
- generating a new node representing the consistency constraint that this cannot be the case (*a no-good node*) or accessing an existing one.
- determining a "loser" from  $b$  and  $\neg b$  to be forced *out*.
- when computing the last step, a culprit in the assumption base for each valid justification for the loser has been determined to be forced *out*.
- for each culprit, effecting reason-maintenance so that all affected beliefs are updated.

BRS combines the technical and logical advantages of a choice of loser as in Doyle's approach with an extralogically supplied ordering of epistemic entrenchment which allows a distinction between candidates involved when revising the belief set.

The main difference between RMS in BRS and the ordinary RMS is that BRS's choice of losers and culprits is based on an ordering of epistemic entrenchment, which is outside the RMS itself. In Doyle's original RMS the losers and culprits is chosen arbitrarily.

## 4.6 Beliefs and inference representation

An assumption is a self-evident piece of information that does not need any justification.

A logic-based formalism is used to represent the beliefs. Formulae that are derived from a set of assumptions will be the believed beliefs. The beliefs are modelled as statements in (a restricted) first-order logic. A meta-theory is used in modelling its deductive process in this logic. There are two logic levels in BRS's formalism — the *object level* and *meta level*.

The reasoner starts from a set of assumptions (self evident beliefs). Only formulae derivable from these are believed. The assumptions are differentiated from other beliefs by the meta-theory. The agent's beliefs are assumed to be closed under the inference system used.

An agent is the owner of the belief. A believed belief has one of its justifications (an assumption) which shows who the believer or agent is. In cases where there is some formula  $f$  and its negation  $\neg f$ , a contradiction will occur. This contradiction will be resolved by choosing a theory that includes only one of them. Decision will be made based on the assumptions' degrees (value) of epistemic entrenchment. The value for each epistemic entrenchment is not within the beliefs' object level logic. The decision is made out of the object level which is at the meta level logic. Hence, reasoning about what is believed on the object level is effected on the meta level. BRS does this in the *BEL*(ieved) predicate. This meta level is consistent. When both  $f$  and  $\neg f$  are *BEL*ieved, a decision rule is invoked to determine which subset of object beliefs to choose.

The following discussion (sections 4.6.1 and 4.6.2) is strongly based on (Berendt, 1992)'s explanation for the BRS' representation of beliefs and inferences.

#### 4.6.1 The object level

The object level is a function-free first-order logic. It has an extended notion of terms as follows:

1. Only constants are taken as terms. ✓
2. Basic formulae are of the form  $p(t_1, t_2, \dots, t_n)$ , where  $p$  is an  $n$ -ary predicate symbol and  $t_i$  are terms.
3. Well-formed formulae are basic formulae, negations of basic formulae, or the form  $A_1 \wedge A_2 \wedge \dots \wedge A_n \supset (\neg)B$ , for basic formulae  $A_1 \dots A_n, B$ .

The standard notion of logical consequence for the object theory is assumed. Only the assumptions from the object level are used directly in reasoning. BRS does not use the object level logic when it reasons about beliefs. It reflects these assumptions and their consequences in terms of the meta language only.

#### 4.6.2 The meta level

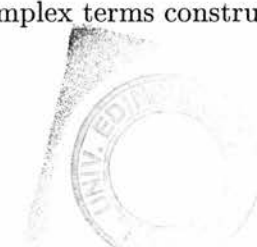
The meta language is a sorted first-order language with equality. The sorts are well-formed formulae of the object language and rational numbers between 0 and 1 as discussed below:

1. Logical constants and logical variables denote well-formed formulae of the object language and rational numbers between 0 and 1.

Every propositional formula of the object language ( $f$ ) is assigned a logical constant ( $\hat{f}$ ), and every rational number between 0 and 1 is assigned a logical constant which is itself.

2. Terms are logical constants and logical variables and complex terms constructed from formulae of the object language as follows:

reduced  
?



- $f \wedge g$  is denoted by *and* ( $\hat{f}, \hat{g}$ )
- $f \supset g$  is denoted by *implies* ( $\hat{f}, \hat{g}$ )
- $\neg f$  is denoted by *not* ( $\hat{f}$ )

3. Well-formed formulae are

- $p(t_1, t_2, \dots, t_n)$ , where  $p$  is an  $n$ -ary predicate symbol and  $t_i$  are terms.
- $\neg f, f \wedge g, f \vee g, f \supset g, f \equiv g, \forall X : f, \exists X : f$ , where  $f$  and  $g$  are well-formed formulae, and  $X$  is a vector of variables free in  $f$ .

Syntax and semantics are defined in the usual way as cf. (Konolige, 1980)

The meta language has several special predicates:

- Assumptions:

$ASSUMPTION(f)$  is true if formula  $f$  is one of the assumptions of the agent, i.e. a belief that is justified by the empty set.

- Tautologies:

$TAUTOLOGY(f)$  is true if formula  $f$  is a constructive tautology.

- Belief:

$BEL(f)$  is true if formula  $f$  is in the agent's belief set.

$BEL$  derives belief in possibly non-atomic formulae of the object language from belief in other such formula. New beliefs are inferred through the following rules:

All assumptions are believed ( $ASS$ ):

$$\forall f [ASSUMPTION(f) \supset BEL(f)]$$

Conjunction (CON):

$$\forall f \forall g [(BEL(f) \wedge BEL(g)) \equiv BEL(f \wedge g)]$$

Implication (IMP):

$$\forall f \forall g [BEL(f \supset g) \supset (BEL(f) \supset BEL(g))]$$

Only beliefs that are justified by these axioms are believed. For any  $\phi$ , the following holds (MINBEL):

$$\begin{aligned}
& \forall f [ASSUMPTION(f) \supset \phi(f)] \wedge \\
& \quad \forall f \forall g [(\phi(f) \wedge \phi(g)) \supset \phi(f \wedge g)] \wedge \\
& \quad \forall f \forall g [\phi(f \supset g) \supset (\phi(f) \supset \phi(g))] \\
& \quad \supset \forall f [BEL(f) \supset \phi(f)]
\end{aligned}$$

*8*

- Epistemic entrenchment:

Degrees of epistemic entrenchment are assigned to object language formulae in the meta language.  $EE(f, e)$  is true if formula  $f$  has the degree of epistemic entrenchment (a rational number between 0 and 1) of  $e$ . The postulates for EE:

- All assumptions and only they have degrees of epistemic entrenchment.
- Only constructive tautologies that are believed can have a degree of epistemic entrenchment of 1.
- All disbelieved statements have a degree of epistemic entrenchment of 0. No believed assumption has a degree of epistemic entrenchment of 0.
- Epistemic entrenchment is functional.
- The ordering must be strict.

- Preference between assumptions on the basis of epistemic entrenchment:

A consistent meta-level theory can represent an inconsistent set of beliefs. This happens if both  $BEL(f)$  and  $BEL(\neg f)$  is believed. One of them,  $f$  or  $\neg f$  has to be dropped from the belief set in order to get a new, consistent set of beliefs. The assumption(s) chosen depends on the overall ordering of epistemic entrenchment.

The fact that BRS is predominantly a foundational system, any evaluation of derived beliefs in these terms must ultimately depend on the evaluation of its underlying assumptions. The choice of a new belief set on formulae derived on this meta-level. These formulae must have assumptions as arguments. This is expressed by the special predicate  $PREFER(a_1, a_2)$ , where  $a_1, a_2$  are assumptions, one of which supports  $f$  and the other  $\neg f$ .  $PREFER(a_1, a_2)$  denotes that the agent prefers to believe the formula  $a_1$  rather than  $a_2$  when resolving an inconsistency between  $f$  and  $\neg f$  which

leads to a contradiction. The belief resting on epistemically more entrenched premises will be the preferred one (of the  $f$  and  $\neg f$ ).

Its detail depend on the decision criterion chosen.

A brief description on the decision criterion is as follows.<sup>11</sup> Consider the decision criterion *maximin* which is formulated as:

An assumption base is a set of assumptions sufficient to derive a belief.

Let the belief  $f$  and  $\neg f$  have the sets of assumption base  $M_1$  and  $M_2$ , respectively.

Let the elements of each assumption base

$m_{ij} \in M_i, i = 1, 2, j = 1, \dots, J_i$  be  $a_{ijk}, k = 1, \dots, K_{ij}$

with degrees of epistemic entrenchment given as  $EE(a_{ijk}, e_{ijk})$ .

Determine  $\hat{a}_{ij}$  such that  $EE(\hat{a}_{ij}, \hat{e}_{ij})$  and  $\hat{e}_{ij} \leq e_{ijk}, k = 1, \dots, K_{ij}$ .

The assumption  $\hat{a}_{ij}$  thus has the minimum degree of epistemic entrenchment in assumption base  $m_{ij}$  and can be regarded as the 'bottleneck' of this assumption base.

Now determine  $\bar{a}_i$  such that  $EE(\bar{a}_i, \bar{e}_i)$  and  $\bar{e}_i \geq \hat{e}_{ij}, j = 1, \dots, J_i$ .

The assumption  $\bar{a}_i$  thus has the maximum degree of epistemic entrenchment among all the bottlenecks in the assumption bases in  $M_i$ .

The decision criterion *maximin* stipulates that the element of the conflicting pair  $f, \neg f$  with higher maximal bottleneck degree of entrenchment remains believed.

This is realised in the system in two steps:

(a) The definition of *PREFER* ensures that

$$\bar{e}_1 > \bar{e}_2 (\bar{e}_2 > \bar{e}_1) \text{ iff } PREFER((\bar{a}_1, \bar{a}_2)(PREFER(\bar{a}_2, \bar{a}_1))).$$

(b) The algorithm for belief changes, *UPDATE* ensures that if

$PREFER(a_1, a_2)(PREFER(a_2, a_1)), \bar{a}_2(\bar{a}_1)$  and thus  $\neg f(f)$  become disbelieved.

Briefly, the algorithm in BRS regarding contradiction is as:

1. Reasoning in the meta-theory to determine if there is a contradiction.

<sup>11</sup> A full description can be obtained in (Berendt, 1992).



2. Reasoning back from the contradiction to determine the assumption bases for the contradictory arguments.
3. Epistemic entrenchment then determines the action to be taken to resolve the conflict.

The algorithm responsible for the belief changes, *UPDATE* will always terminate and lead to a new non-absurd belief state.<sup>12</sup> Contractions will only be effected if the tentative belief set is inconsistent. This algorithm is guaranteed to lead to a consistent belief system while assembling as much information as possible. The construction of belief sets from assumptions via the axioms for *BEL* ensures that every belief is justified. At the same time, belief changes observe the principle of conservatism and the principle of maximum epistemic entrenchment. In the former principle, as many as possible of the existing beliefs are to be retained. In the principle of maximum epistemic entrenchment, the preferred subset (if there is one) is to be retained.

## 4.7 The Belief Revision System's features

Berendt's work has combined the strength of both the RMS and AGM theories by developing a system that keeps a belief set consistent and able to justify and explain all its beliefs while making choices to resolve conflicts on the basis of epistemic entrenchment.

The properties of the developed BRS in terms of its ability are as follows:

1. Reason about beliefs and derive new beliefs from given assumptions.
2. Exhibit the beliefs and their justifications involved in the reasoning, for explanation.
3. Detect contradictions and resolve them, thus maintaining the consistency of the knowledge base.
4. Allow to make choices on how to resolve a contradiction on the basis of epistemic entrenchment.

---

<sup>12</sup> A set of statements which is closed under the respective notion of logical consequence and consistent.

5. Allow the user to experiment with different ordering of entrenchment on the beliefs.

#### 4.8 BRILM - the belief revision in TRAPS' learner modelling

TRAPS' educational aim is to assist its learner modelling to elicit a learner's entrenched contradictory schemes and utilise the information in the remediation session.

*clearly*  
TRAPS' particular focus on a domain i.e committing reversal error in relational algebra word problems, portrays the characteristic of contradictions of beliefs in the learner's reasoning process. If such a contradiction of beliefs is sorted out or resolved, this could result in consistent beliefs in the learner's reasoning. In other words, if a contradiction of beliefs occurs in the knowledge base, this signifies the existence of the reversal error. *why?*  
If the beliefs in the knowledge base are consistent, the learner's reasoning does not exhibit the reversal error. Thus, TRAPS main approach of detecting the reversal error and misconceptions leading to the reversal error is through the occurrence of contradiction of beliefs in a knowledge base. Resolving the conflicting beliefs is TRAPS' attempt to remove the reversal error and its related misconceptions.

Due to such an approach, BRS is found to be a suitable mechanism in TRAPS' learner modelling. BRS's combination of the foundational and coherence approach in dealing with conflicting beliefs assists TRAPS in dealing with the learner's contradictory schemes which leads to a reversal error in relational algebra word problems.

TRAPS employs such a belief revision system to resolve the two levels of conflicts that arise when the student formulates the reversed equation from an algebraic word problem.

We are interested in utilising BRS's capability to:

1. Reason about beliefs and derive new beliefs from given assumptions for TRAPS to record TRAPS' beliefs, the learner's beliefs and TRAPS' learner modelling beliefs.
2. Detect conflicting beliefs for TRAPS to uncover the learner's contradictory schemes.



3. Resolve the contradictions for TRAPS to inform the learner of the correct belief (one of the two beliefs which contradict each other).
4. Explain the reasoning of the involved beliefs and justifications for the learner's remediation session in TRAPS' LTB-explanation<sup>13</sup>.
5. Choose how to resolve contradictions on the basis of epistemic entrenchment for TRAPS to decide whose beliefs (those of TRAPS' or the learner's) in Level-1 conflict, and which of TRAPS' beliefs in Level-2 conflict.

However some modification of the original BRS are made to suit TRAPS, as a tutoring system. The modification made are as follows:

1. We have to violate the principle of conservatism. BRS applies conservatism when two beliefs of the same entrenchment value are to be revised. In TRAPS, such a situation arises when TRAPS makes its meta level observations on the student's performance. Here, a conflict between TRAPS' own beliefs may arise. The competition of the beliefs is between two of TRAPS' own entrenchment values (*Tous*) where both have the same value. Berendt's system applies Gärdenfors' *conservative* way of choosing the belief that is to be kept when resolving a conflict: the earlier belief is considered to be more reliable compared with the new ones. Unfortunately, this is not the case during a teaching session. An earlier observation should not necessarily be assumed to be more valid than a recent one. In TRAPS, we evaluate the abstract properties of the questions involved and refer these entities to the related *Tous*. Among the entities evaluated for each question are:

*This is not conservatism.*

- its level of difficulty;
- whether it has been answered correctly or not by the student; and
- the concept it carries (eg. the understanding of the coefficient's function in the equation; or mapping the keywords of the problem statement).

<sup>13</sup> This is described further in section 5.5.2 and section 6.7.1

The *Tov* that has weaker justifications out of this evaluation will have its entrenchment value decremented and thus be dropped from the belief set. The underlying mechanism for this evaluation will be further described in Chapter 6.

2. BRS gives the freedom to the programmer and user to enter any valid entrenchment value (any number within 0 and 1). We have however, fixed the *qualitative value* of the entrenchment value. The fixed qualitative value refers to the fact that any of TRAPS' beliefs ( $B_T$ ) has to be more entrenched than the learners' beliefs ( $B_L$ ). The entrenchment value assigned can be any of the valid number as long as it obeys our fixed qualitative value. As a tutoring system, we assumed that  $B_T$  is usually more constructive and reliable compared to that of  $B_L$ .<sup>14</sup>
3. Related to the second amendment above, we redesigned the system in such a way not to allow the programmer and user to test different combinations of epistemic entrenchment values. TRAPS' tutoring strategy is not the kind that allows the learner to experiment with their answers and learn from the experiment's results. Even if the epistemic entrenchment value can be defined as the "strength of validity" of a belief, allowing the learner to test the combination of these values is not within our scope of tutoring strategy. TRAPS' tutoring strategy is more designed to elicit the learner's contradictory beliefs (which causes the reversal error he commits) through a series of entrapping questions.

*C is this a fun?*

## 4.9 Some limitations of the revision systems

There are two outstanding limitations in BRS. We discuss them as follows:

1. RMS can either be in a *single-context*<sup>15</sup> (maintaining one belief set at a time) or *multiple-context*<sup>16</sup> (maintaining multiple belief sets at a time). Berendt's research concentrates on one consistent system of beliefs, not a set of possible, contradicting justifications for different consistent systems of belief. In other

<sup>14</sup> This may be a rigid assumption. Unless our tutoring approach deals with negotiation, we consider this assumption appropriate and assumptions other than this is beyond the scope of our learner modelling.

<sup>15</sup> Usually identified by JTMS (Justification Truth Maintenance) (Doyle, 1979).

<sup>16</sup> Identified by ATMS (Assumption Truth Maintenance) (De Kleer, 1986).

words, BRS is applicable only for maintaining the consistency of a single system of beliefs.

This is however does not influence BRILM because TRAPS learner modelling approach deals with consistency of the learner's beliefs and that of TRAPS' beliefs which are both grouped together in **one shared belief set**. The grouping into one belief set is because both agents concentrate in resolving a single word problem, at a time. We are interested in maintaining the internal consistency of reasoning for both the learner's and TRAPS' beliefs when combined together.

2. There is no differentiation of beliefs in terms of their agents. This is to simplify the logic in BRS (Berendt & Smaill, 1992). However, differentiation of agents could be implemented by having a second argument in the meta language predicates. This argument will denote the theory (the reasoning or part of the reasoning of an agent) referred to.

For the purpose of TRAPS' modelling, the current meta language is sufficient for TRAPS to determine the agent of the belief. This is done by using different assumption for different agents, such as below:

`add_assumption(says(traps, X) → X, Tev).`

`add_assumption(answers(student, X) → X, Sev).`

I need to explain.

Despite these two limitations, the first one does not affect BRILM's application in TRAPS and the second constraint is overcome by TRAPS distinguishing the agents in its rules.

## 4.10 Conclusion

This chapter has described Berendt's belief revision system BRS, which has then been modified to BRILM. The description focuses on the systems' underlying theory and mechanisms of realising TRAPS' learner modelling goals.

Further discussion on BRILM being used as the basis for maintaining the internal reasoning consistency in TRAPS learner modelling as an approach to diagnosing and

remediating the learners' *contradictory schemes*<sup>17</sup> is in the following chapter.

---

<sup>17</sup> This is defined in section 3.6.

## Chapter 5

# Handling entrenched misconceptions and contradictions

This chapter begins with discussion about the ways in which people deal with information that contradict their entrenched beliefs. In relation to relational algebra word problems, we describe TRAPS' instructional design that is meant as an approach to elicit and handle learners' entrenched misconceptions.

This is then followed by a description of the characteristic of contradictions that occur when a learner commits a reversal error within a problem set<sup>1</sup> (Level-1 conflict), and contradictions that could occur across problem sets (Level-2 conflict). The former conflict level reveals the *conflicting schemes*<sup>2</sup> which exists in the learner's reasoning, while the latter conflict level exhibits the learner's entrenched misconceptions.

The discussion proceeds with the idea of **modelling through cognitive conflict** which suits TRAPS approach of handling Level-1 conflict. In relation to this, the description on how TRAPS models the reversal error is divided into **diagnosing** and **remediating** the error, and its misconceptions are explained. However, modelling Level-2 conflict is a bit different from that of Level-1. Discussion on modelling Level-2 conflict in TRAPS is presented before ending this chapter.

---

<sup>1</sup> A number of related questions concerning a particular relational word problem.

<sup>2</sup> This is defined in section 3.6.

acknowledging (why not?)

## 5.1 Dealing with entrenched beliefs

**Entrenched beliefs** are defined as *beliefs that are strongly bound in a web of observational and/or theoretical support* (Brewer & Chinn, 1991). In relation to this, we define **entrenched misconceptions** as *incorrect conceptual understanding or beliefs that are deeply seated, tenacious, resilient, and not easily dislodged from the believer's reasoning*. Such entrenched misconceptions seem to be useful (although incorrect) to the believer in his reasoning process and argumentation.

In order to propose a method for assisting learners to mend their entrenched misconceptions, we refer to studies that describe how people handle entrenched beliefs. (Brewer & Chinn, 1991)'s study illustrates how people deal with contradictory information which at the same time violates their entrenched beliefs. In the experiment, Brewer & Chinn provided new information that contradicted the subjects' entrenched beliefs. It was found that the individuals responded to such a situation by:

**BC1** - rejecting the information; or

**BC2** - reinterpreting the information; or

**BC3** - making partial changes in their existing beliefs through accommodation and/or compartmentalization; or

**BC4** - restructuring their incorrect beliefs.

In BC1, some of the subjects tend to refuse to accept the contradictory information and persist in believing the information they have known before. As for those in BC2, they reinterpret the contradictory information to make it consistent with their existing beliefs. The new information seems not to have an impact on their entrenched beliefs, but is interpreted according to their existing knowledge and beliefs. In BC3 situation, the subjects' entrenched beliefs are partially modified. Their prior beliefs are changed by accommodation and/or compartmentalization. In the accommodation approach, the individuals attempt to incorporate the new information into existing knowledge while preserving the core beliefs of their existing knowledge. On the other hand, modification by compartmentalization is an approach where the subjects believe some

parts of the new information and part of their entrenched beliefs. The last response as in BC4, people may react to information that contradicts their tenacious beliefs by accepting the new information completely i.e restructuring their existing knowledge or beliefs. However, such a situation does not happen easily.

In other work, Schauble investigates how children revise their knowledge over an extended period of time in the light of new evidence that they actively create and interpret. Among discussion he made on the result of his experiment:

*“At times children appeared to overrely either on their theories or on outcomes of the experiments. Children who overrelied on their (prior) beliefs substantially misread the evidence, misinterpreting or distorting it so that they could conclude that it supported their favoured theories.”*

(Schauble, 1990), p. 55

Hence, to revise an existing belief or knowledge is an action that many people seem reluctant to make. This is even true when they are to revise their entrenched beliefs and knowledge.

In relation to relational algebra word problems, (Rosnick & Clement, 1981) have listed evidence of behaviour that illustrates the entrenched misconceptions which contributes to the learner's reversal error:

**RC1** - the learner reverts back to the reversed equation;

**RC2** - the learner accepts the correct equation but switches the meaning of the original problem;

**RC3** - the learner identifies the correct equation as being “weird” or “not making sense”; and

**RC4** - the learner acknowledges that the correct equation “works” but states that he doesn't know why it works.

TRAPS combines both (Rosnick & Clement, 1981)'s evidence and (Brewer & Chinn, 1991)'s findings when designing questions for the learners. This is to elicit or diagnose



T: Jean wants to prepare a home-made salad dressing.  
According to the recipe, she has to mix 3 times  
as much oil as vinegar.  
Does Jean has to put more oil or vinegar?

- a. vinegar      b. oil

L: b

T: Try formulate an algebraic equation of the  
statement "There is 3 times as much oil as vinegar  
in the salad dressing", using the letter L for the  
amount of oil and V for the amount of vinegar.

- a.  $3L = V$       b.  $3V = L$

L: b

T: According to the equation  $3V = L$ , if Jean has put  
a cup of oil, she will have to mix it with 3 cups  
of vinegar.  
Do you agree with that? Yes/No (y/n) ?

L: y

---

Figure 5.1: A sample of the diagnosing session in TRAPS.

entrenched misconceptions that may exist in the learners' reasoning. The integration is as follows:

(RC1 & BC1) - accept the correct equation temporarily and reject it by accepting the reversed equation at a later stage;

(RC2 & BC2) - accept the correct equation but reverse the meaning of the relational statement (the original problem);

(RC3 & BC1) - refuse to accept the correct equation and/or correct translation strategies;

(RC4 & BC3) - believe partially the correct equation and/or correct translation strategies to be correct but may tend to commit the reversal error in entrapment questions;

Figure 5.1 is an example of (RC2 & BC2), where the learner formulates the correct equation but in the next question, he interprets the meaning of the original problem as its reversed, i.e. validating the reversed equation unconsciously.

## 5.2 Instructional design in TRAPS

Instruction is the presentation of the subject material to the learner. Different instructional objectives <sup>we</sup> have different tutorial dialogue strategies. (Halff, 1988) summarized Collins and Stevenson's guidelines for tutorial dialogues as in Table 5.1

Considering the characteristics of the reversal error in relational algebra word problems, we find that all three instructional objectives in Table 5.1 are appropriate for TRAPS. We incorporate the instructional approaches of **eliciting a fact or concept, entrapment, teaching the correct rules, and giving questions oriented to concepts which the learner has problem with.**

There are two parts to the instructional design of TRAPS:

1. the questions - for the Pre-test, *Teaching and learning* session and Post-test.

Instructional Objectives	Strategies
Teach facts and concepts Explain fact or concept	Elicit fact or concept Teach rules and relations Case selection strategies Entrapment
Teach induction skills	Exercises and examples oriented to subskills

Table 5.1: Instructional objectives and the related dialogue strategies (Halff, 1988).

2. the explanation - for the *Teaching and learning* session and the *observation conflict level*.

In this section, discussion is focused on the questions' design and implementation. The design for explanation content is described in section 5.5.2. The questions are designed in TRAPS based on empirical studies of (Clement et al., 1979), (Rosnick & Clement, 1981), and that of (Brewer & Chinn, 1991). This approach is applied in order to accommodate common anticipated correct and incorrect learners' behaviour and solutions. The approach incorporated in designing the questions are:

1. Entrapment - integration of (Rosnick & Clement, 1981)'s and (Brewer & Chinn, 1991)'s findings as described at the end of section 5.1.
2. Cascaded sets of problems<sup>3</sup> with of *layers of difficulties*.

The aspects of difficulties that have been considered:

- using variables that are not the first letter of the objects (eg. using X and Y instead of S and P in the *student-professor* word problem);
- use divisional equation instead of multiplication equation;
- pose word problems for which their qualitative values are not easily guessed through common sense of daily examples (e.g the number of planes in a region);

These aspects attempt to elicit the entrenched misconceptions that learners might have. Applying such concepts on *layers of difficulties* seems to be an appropriate

---

<sup>3</sup> Full description is in section 6.3

approach for this. As noted by Twidale in relation to his evaluation on the EPIC system,

*“... by encouraging articulation of beliefs and focusing on the understanding of a few students, the EPIC study revealed that in certain cases, students may be able to competently perform simple exercises in a domain while still having major misconceptions about that domain, that only become apparent when they tackle a harder problem ...”*

(Twidale, 1993), p. 174

3. Multiple choice answers which includes reasons for each of the answers, incorporating items which capture common incorrect answers and misconceptions. As mentioned by (Murray, Schultz, Brown, & Clement, 1990), the reasons for the learner's answers are just as important as the correctness of the answer.
4. Hidden multiple choice questions such as requesting the learner to either agree or disagree a statement by answering Yes or No.
5. Provoking questions such as the last question in Figure 5.1.
6. Questions which are isomorphic are asked repeatedly to ensure that their incorrect answers are not slips.
7. In some questions, TRAPS provide pertinent information such as knowledge about the mathematical notation (eg. *C is the number of children in the kindergarten*), and hints about visualising the problem (eg. *Given that there are 10 teachers, how many children are there?*) for the learner to apply in his reasoning process while translating and formulating the algebra word problems. (Bernado & Okagaki, 1994) describe the information on the mathematical notation as *symbolic knowledge* and the hints as *problem-information context (PIC)*. (Bernado & Okagaki, 1994) also found that learners are more likely to construct correct equations with the symbolic knowledge present, and those who received appropriate PIC did better in translating relational algebra word problems and constructing algebraic equations.
8. In terms of TRAPS-learners interaction, we apply the delay feedback. This happens in the pre and Post-test sessions and early part of the questions in the

teaching and learning session. Here, TRAPS withholds revealing the correct answers and gives no indication of whether the answer is correct or not. This strategy makes the learners ~~to~~ emphasis ~~on~~ thinking about the questions and reason for their answers. The instructional methodology of withholding substantive feedback from learners has been found to be a successful approach of teaching mathematics and science in many tutoring situations (Lochhead, 1983).

The questions are organised according to the structure of the *Cascaded Problem Set* (CPSA) teaching architecture (Schank, 1990). Under such an architecture, a library which is filled with questions are organised in several sets of word problems. Each set is decomposed into constituent parts, i.e several questions related to a particular word problem. Each question has its own underlying concept<sup>4</sup>. The overall picture of the underlying concepts for the questions are in Figure 5.7 and Figure 5.8. Further description of implementing CPSA is in section 6.3.

### 5.3 Characteristics of the contradictions

In TRAPS, we assume<sup>5</sup> that a learner commits a reversal error when one or more of the following happens:

1. he formulates a reversed equation;
2. he agrees with a given reversed equation;
3. he does not formulate a correct equation;
4. he rejects a correct equation;
5. he agrees with the contradictory schemes;
6. he applies any of the incorrect translation strategies.

The reversal error in relational algebra word problems is basically due to existing contradictory schemes in the learner's reasoning, resulting in a conflict of beliefs. We

---

<sup>4</sup> These concepts will be discussed in section 6.3.

<sup>5</sup> Based on analysis of empirical studies as discussed in chapter 3.

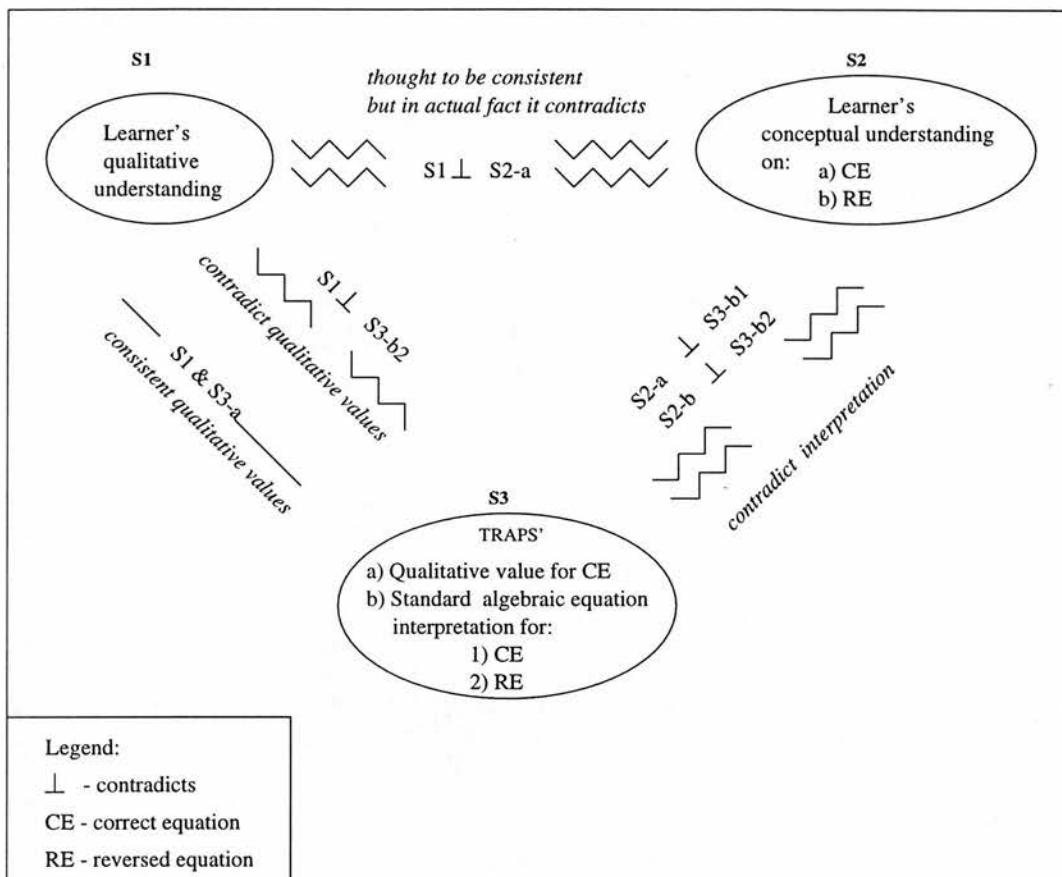


Figure 5.2: Level-1 conflict - the *triangle conflict* that occurs when learner commits the reversal error.

define a conflict as believing both a concept  $C$  and  $\neg C$  at the same time or in other words a contradiction of beliefs. The terms conflict and contradiction refers to the same meaning in our discussion.

When a learner commits the reversal error, TRAPS seeks to derive contradictions which take place:

1. within the learner's own beliefs;
2. between the learner's and TRAPS's beliefs about the domain; and
3. among TRAPS's own beliefs about the learner.

The contradictions can be seen at two levels — **Level-1 conflict** and **Level-2 conflict**. The following discussion explains both types of contradictions.

In the **Level-1 conflict**, contradictions occur **within the learner's reasoning process**, that is between his *qualitative understanding*<sup>6</sup> and his *conceptual understanding*<sup>7</sup> of the correct equation. For example, in the sentence "There are 5 times as many palm trees as coconut trees that dad had grown on the piece of land", a learner believes qualitatively that there are more palm trees than coconut trees. His interpretation of the correct equation<sup>8</sup>  $5C = P$ , ( $P$  as the number of palm trees and  $C$  as the number of coconut trees) implies the opposite situation i.e there are more coconut trees than palm trees. This contradiction is between the learner's qualitative understanding and that of his incorrect interpretation of the correct equation. This is described in Figure 5.2 as  $S1$  contradicts  $S2-a$  which can be indicated as  $S1 \perp S2 - a$ . At the same time, the learner's qualitative understanding contradicts TRAPS' interpretation of the reversed equation i.e  $S1 \perp S3 - b2$ . However, the learner's qualitative understanding and that of TRAPS' qualitative value, is consistent. i.e both the learner and TRAPS believe that there are more palm trees than coconut trees in the example above.

In this level of conflict, a contradiction also occur **between the learner's interpretation of the equations** (both the reversed and correct ones) with that of **TRAPS' own implications of both equations according to the standard algebra interpretation**. As illustrated in Figure 5.2,  $S2a$  contradicts with  $S3-b1$  and  $S2-b$  contradicts with  $S3-b2$ , indicated as  $(S2 - a \perp S3 - b1)$  for the former and  $(S2 - b \perp S3 - b2)$  for the latter.

These contradictions that could arise at Level-1 conflict and illustrated in Figure 5.2 is also referred to as the **triangle conflict**.

This conflict level emerges during the diagnostic stage which takes place in the teaching session. The beliefs and justifications involved in the belief revision process derived here can be used to remediate the learner's misconceptions for the particular problem he is translating.

In TRAPS' **Level-2 conflict**, the learner's beliefs (the misconceptions) ~~is~~<sup>are</sup> seen to be deeply entrenched in his reasoning. The learner may fail to be consistent over

<sup>6</sup> Knowledge on which of the groups in the relational statement is larger and smaller in number.

<sup>7</sup> Interpretation of equation that is translated from the relational statement.

<sup>8</sup> Although he may not know that this is the correct equation.

in what sense  
is this a conflict?



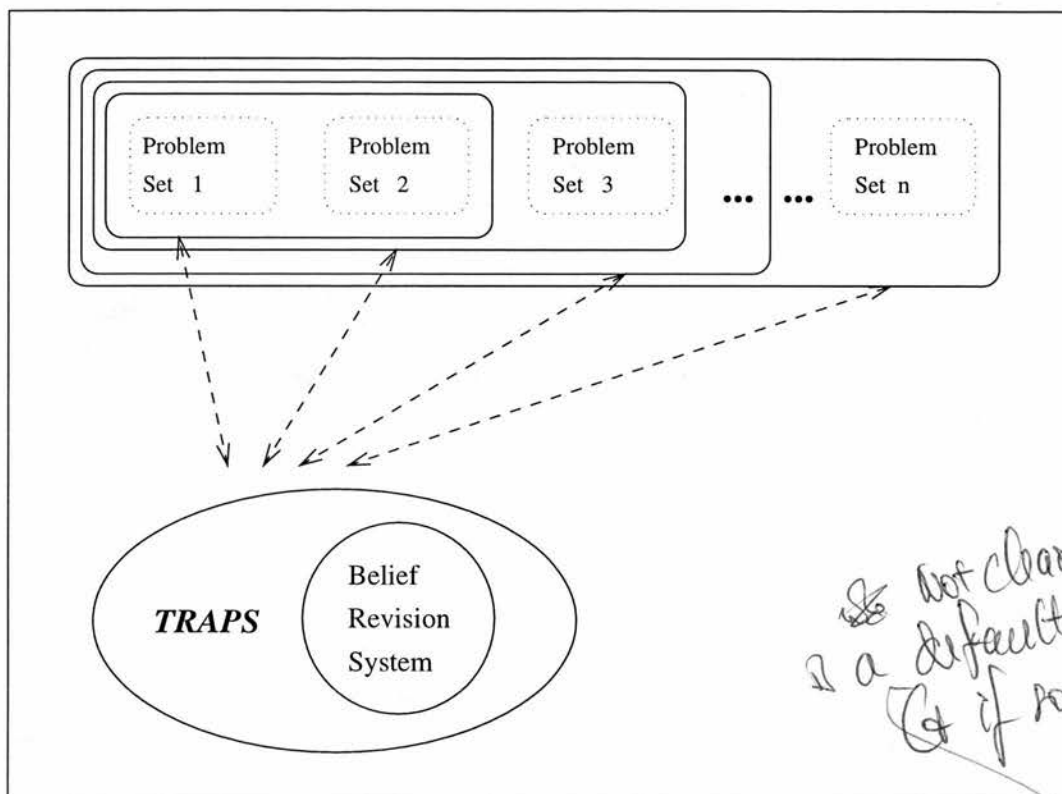


Figure 5.3: Level 2 conflict - TRAPS's meta level observation on the learner's performance.

time due to the resurfacing of his tenacious misconceptions. This causes TRAPS's beliefs that *"the learner is capable of translating relational algebra word problems"* to contradict *"the learner is not capable of translating relational algebra word problems"*. Here, TRAPS's belief about the learner's capability contradicts with another belief of TRAPS about the learner's performance, over time. TRAPS' observation compares the learner's performance in the current set of problems he is doing with his performance in the previous problem sets he had gone through as indicated by the solid boxes in Figure 5.3. These entrenched misconceptions may be the same ones or different ones at each stage of observation. Such a conflict may occur when TRAPS evaluates the learner's performance across problem sets. This can be referred to as TRAPS' **observation conflict**.

Level 2 is the meta level observation by TRAPS on the learner's overall performance. The learner's record on level 2 conflict is recorded in a file that could be retrieved by the teacher after the online session. It is meant as reference to assist the teacher in

assessing the learner's overall performance.

As mentioned before, Level-1 conflict occurs within one problem set, whereas Level-2 occurs across problem sets. The former conflict level reveals that conflicting schemes exist in the learner's reasoning process. On the other hand the occurrence of Level-2 conflict is an indication of resurfacing of misconceptions which suggests the existence of entrenched misconceptions in the learner's reasoning process. Both level<sup>s</sup> of contradictions require going back down the chain of reasoning and deciding on the relative merits of beliefs that underlies the conflicting beliefs. This is the process of **resolving the contradictions**. Resolving contradictions assist<sup>s</sup> TRAPS' remediation session for the learner, by providing the beliefs (information)<sup>s</sup> involved in the conflict. More will be discussed in section 5.5.2.

#### 5.4 Modelling through cognitive conflict

The idea of our learner modelling falls in the category of modelling through cognitive conflict. In this technique TRAPS **diagnose through contradiction of beliefs and assisting the remediation process by using the beliefs and justifications involved when resolving the diagnosed contradictions**. *s. or pl.?*

The basis for the diagnosing technique in the domain of the reversal error in relational algebra word problems is the **existence or absence of a contradiction**. If a contradiction occur<sup>s</sup>, then misconception is assumed to exist within the learner's reasoning. On the other hand if there is no contradiction, then no misconception is assumed to occur in the learner's reasoning. As for assisting the learners to mend their misconceptions, TRAPS utilise<sup>s</sup> the beliefs (and its logical flow) that are responsible for the contradictions and the beliefs involved when resolving the conflict. These beliefs form the contents for the explanation given to the learners in the triangle conflict, and to the teachers in the observation conflict. Figure 5.4 summarises the learner modelling technique we apply in TRAPS.

Such a technique is seen to be parallel to Self's discussion of approaches that may be involved in *Computational Mathematics*<sup>9</sup>:

---

<sup>9</sup> Defined as the study of learning, and how it may be promoted, using the techniques, concepts and

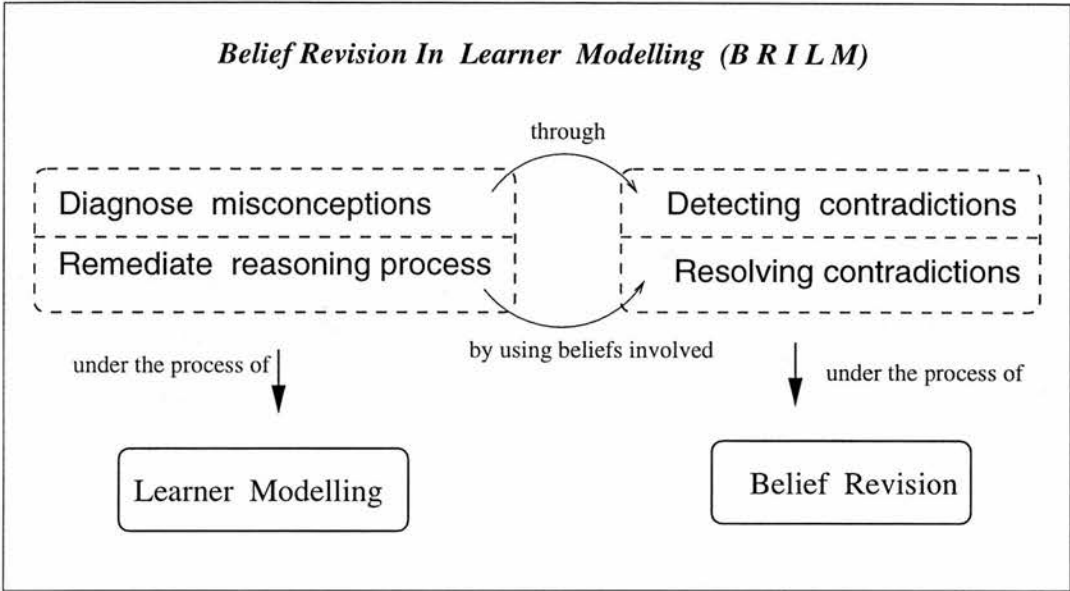


Figure 5.4: The learner modelling technique in TRAPS.

**“Learning through cognitive conflict** - *A prevailing view of learning is that it is provoked by a conflict between the agent’s beliefs and some acquired evidence. This conflict may be deliberately created by some other agents, e.g. a teacher or ITS, or sought by the learner. Computational Mathematics needs therefore to consider formally how conflicts may be created, and their effects on a learner. This will involve some consideration of philosophical views of belief change and the application of AI work on reason maintenance.*”

(Self, 1991), p. 6

### 5.5 Modelling the reversal error

This section will discuss TRAPS’ modelling applied to the domain of relational algebra word problems. We refer to modelling the learner as the diagnose process while mending the learner’s misconceptions as the treatment or remediation process. TRAPS’ overall learner modelling involves the diagnosing process and the remediation process. An overview of both modelling and mending process in TRAPS is as in Figure 5.5.

---

methodologies of computer science and artificial intelligence (Self, 1991).

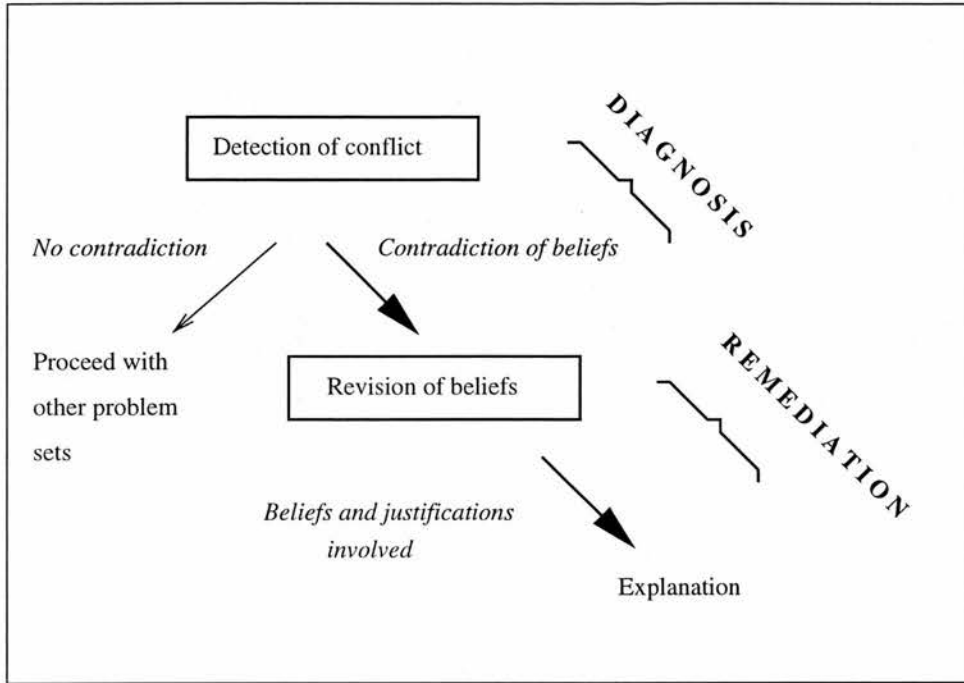


Figure 5.5: Diagnose and remediation in TRAPS' learner modelling.

### 5.5.1 Diagnosing the misconceptions

A learner's existing knowledge may influence his reasoning. Hence, it is important to diagnose his prior knowledge or beliefs. Diagnosing the learner is meant to uncover his hidden cognitive states from his observable behaviour or belief on the subject matter. For this, the diagnosing process infers the learner model.

Diagnosing is based on the characteristics of the contradiction and misconceptions leading to the conflict of belief, as discussed in earlier sections. The diagnosing approach we have taken can be viewed in Figure 5.6. Before the learner begins to answer a problem-set, TRAPS generates all relevant facts related to the problem-set as TRAPS' own beliefs  $B_T$ . Then questions in the problem-set are displayed for the learner to answer. The learner's answers are taken as the learner's beliefs,  $B_L$ . TRAPS will then interpret  $B_L$  accordingly which produces part of the Learner Model's beliefs,  $B_{LM}$ .

All these beliefs —  $B_L$ ,  $B_{LM}$  and  $B_T$ , are inferred in BRILM to check for consistency as the diagnosing approach. If a conflict of beliefs is encountered, then there exist

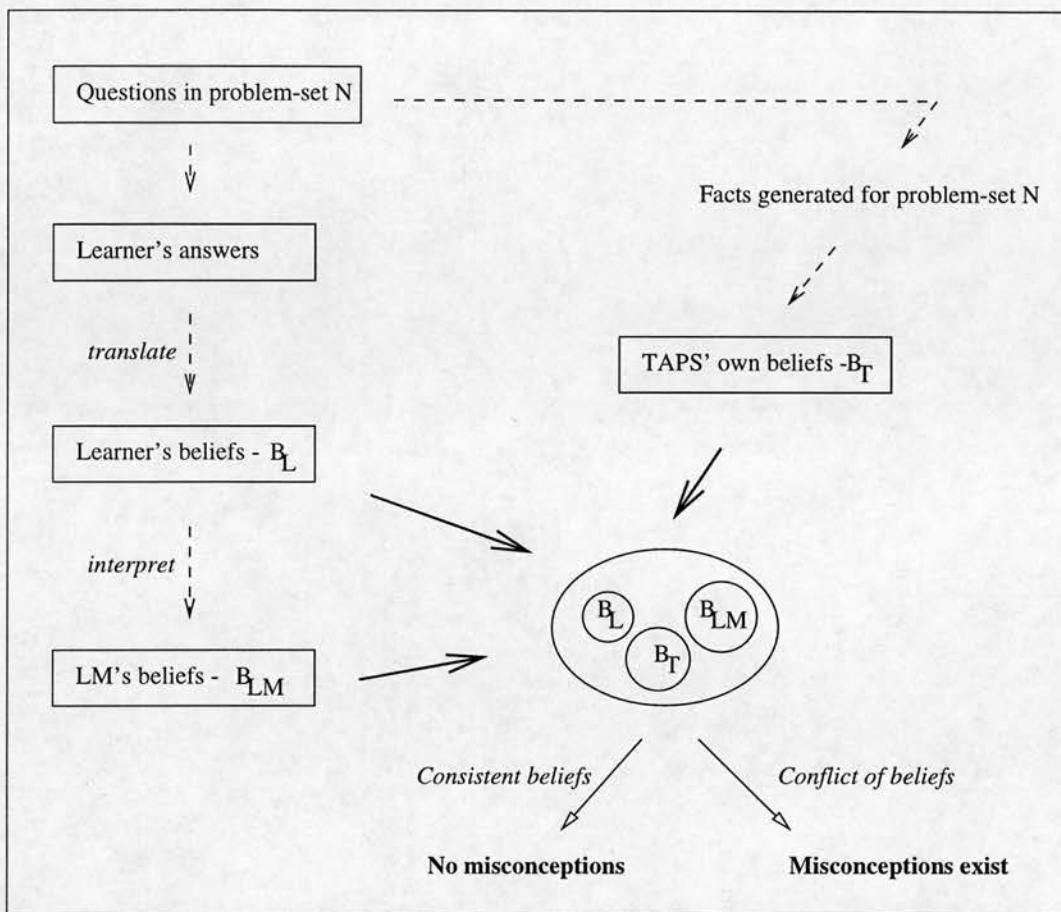


Figure 5.6: Diagnosing a learner's errors and misconceptions within one problem set.

misconceptions related to the reversal error.

*why is no other error considered?*

### 5.5.2 Remediating the misconceptions

The emphasis of design in most ITSs is to promote learning by providing new knowledge and remediation of what are considered to be errors with respect to the domain's correct representation. Errors<sup>10</sup> are due to learners' misconceptions. In *learning from mistakes*, errors are valuable for learners. From their errors or mistakes, and especially from traces of their own mistakes, learners learn to identify their own misconceptions and to see how such misconceptions could have arisen. We incorporate the above idea in TRAPS, when assisting the learners to mend their misconceptions and accept new knowledge to correct their conceptual understanding. This happens in TRAPS'

<sup>10</sup> Particularly the reversal error in TRAPS

remediation session.

Two forms of explanation are generated in the remediation session:

- Explanation generated from the learner's and TRAPS' specific beliefs which involve in the contradiction. We refer this as the **LTB-explanation** (Learner and TRAPS Beliefs explanation). The reasoning that causes the contradiction is presented by displaying the beliefs (belong to the learner and TRAPS) and their implications in a logical flow. The logical flow shows the steps, justifications and consequences of the beliefs that contradict. An example of LTB-explanation is in Figure 6.14.
- Explanation generated from the partial canned-text catering for the specific problem sets and the related translation strategy which the learner currently applies and the correct approach TRAPS has. This type of explanation is referred to the **PCT-explanation** (Partial Canned-Text explanation). PCT-explanations emphasises the strategies of translation ~~in its explanation~~. An example of PCT-explanation is in Figure 6.16.

Implementation <sup>f</sup> of both types of explanation is described in section 6.7, and the usage of the LTB and PCT explanation is further discussed in sections 7.4.4 and 7.5.4.

## 5.6 Modelling the observation conflict

We have mentioned TRAPS' meta-level observation of the learner's performance in the Level-2 conflict (observation conflict). In such observations contradiction of beliefs could occur too. Contradictions of beliefs in the Level-2 conflict are recorded off-line in a file for the teacher's reference. The outcome of the teaching and learning session is whether the learner is capable or not capable of translating relational algebra word problems. The pattern of occurrence of such conflicts could be used to determine whether the particular learner has entrenched misconceptions. If there exist such deep seated misconceptions, the information from this level of conflict could <sup>indicate</sup> tell the type of resurfacing misconceptions. Further discussion on generating this explanation for the teacher is discussed in section 6.8.

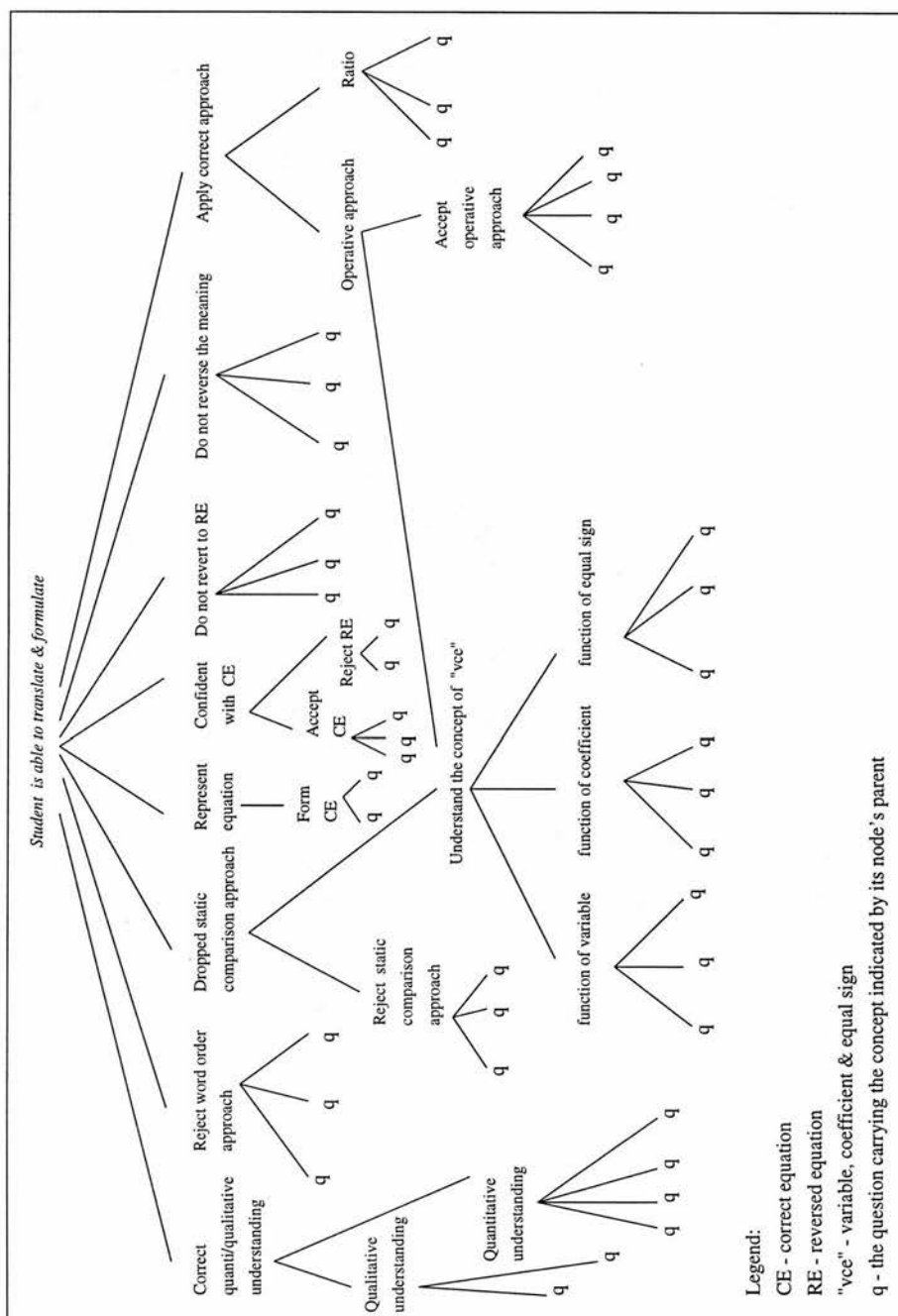
The judgement of whether the learner is capable or not capable of translating relational algebra word problems is partially based on the trees in Figures 5.6 and 5.6. The trees in these figures are designed based on empirical findings on learners' behaviour when translating algebra word problems (Rosnick & Clement, 1981). The detail underlying mechanism for resolving the Level-2 conflict is discussed in section 6.6.

## 5.7 Conclusion

In this chapter, we have discussed TRAPS' learner modelling technique. This has been described by explaining the approach TRAPS applies to diagnose entrenched misconceptions and contradictions related to the reversal error. Explanation of TRAPS' method of remediating learners' misconception has also been presented. Modelling contradictions that may occur during TRAPS' observation of the learners' performance has also been covered in this chapter.

The following chapter continues the discussion of this chapter by describing the implementation of the question library (for the Cascaded problem set teaching architecture) and TRAPS' learner modelling.





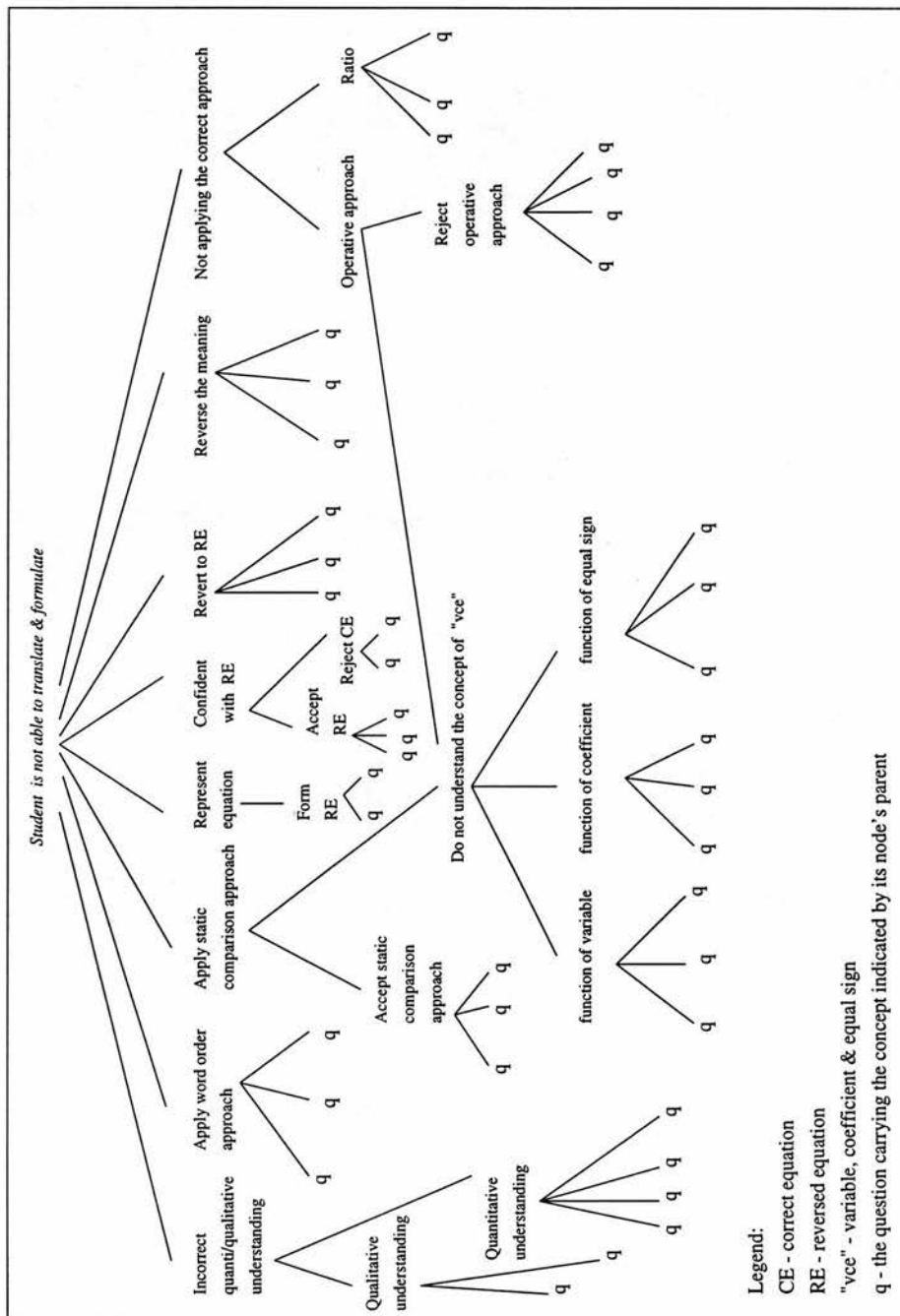


Figure 5.8: Questions to identify if the learner is not able to translate relational algebra word problems

## Chapter 6

# Implementation

This chapter discusses the implementation of TRAPS. The discussion is based on the components that TRAPS has and presented in the sequence in which they are implemented. We go through the stages of implementation in TRAPS first. Then we describe the overall interaction session between TRAPS and the learner. The discussion on the design and implementation begins to get deeper by describing our pre-defined question library, its structure and contents. We then discuss the generation of beliefs, adding and updating them in our *BRILM* (**B**elief **R**evision **I**n **L**earner **M**odelling). We highlight and describe our approach for handling contradicting beliefs in *BRILM*. Following this, the explanation generator and learner's record generator are described.

### 6.1 Stages of implementation

TRAPS' design and implementation starts off with the compilation of questions of word problems. The questions are designed and organised according to the *Cascaded Problem Set teaching architecture*.

We then generate the beliefs involved for each word problem — from the relational statement, from the possible answers given by the learners and from TRAPS' implication of the learner's answers. This involves building a simple sentence parser and some *if-then* rules.

This leads us to interface TRAPS with *BRILM*. Beliefs are added and updated in *BRILM*. If there is any contradiction, some of the beliefs will be dropped from the

BRILM's belief set and others may remain.

We then proceed with the explanation component which relies on the information passed by BRILM. Two types of explanation are generated for the learner's remediation session. At the same time the learner's performance record can also be produced.

## 6.2 Interacting with TRAPS

There are three stages in the learner's interaction with TRAPS:

- Pre-test;
- teaching and learning session;
- Post-test.

In each session, the questions in each stages are presented in a multiple choice form. This is to make it easier for the learner as only a single character is needed as their answer for each question.

### 6.2.1 Pre-test session

The pre-test comprises problems designed to elicit the misconceptions related to the reversal error. Questions are given and no feedback on the correctness of the answers are provided at this stage.

It is designed to:

- determine the level of the learner's translation capability: below average, average, above average;
- identify the learner's strategy in translating the problem;
- discover the learner's misconceptions; and
- assist in the evaluation of the learner's progress i.e. as a comparison to his post-test performance.

In relation to the three mastery levels identified, TRAPS is designed to concentrate on the middle or **average mastery level** where the learner is partly correct in his translation process and partly incorrect. The learner at this level knows that it is a multiplication equation that he has to formulate but his misconceptions lead him to formulate a reversed multiplication equation. We believe that *contradictory schemes*<sup>1</sup> and inconsistent performance occurs more among average level learners. It is at this level, that the usage of belief revision is needed more.

### 6.2.2 Post-test session

The post-test given after the teaching session is of a form isomorphic to the pre-test, and is used primarily for assessment of the learner's performance. The results are compared<sup>2</sup> to the pre-test performance in order to evaluate whether the teaching session has played a positive role in assisting the learner.

### 6.2.3 Teaching and learning session

Between the Pre-test and Post-test sessions, a more interactive session called the *teaching and learning* session (TL session) is held. The contents are based on the learner's mastery level. The session is in a multiple choice question-answer *dialogue* format. The underlying methodology of the series of questions posed to the learner has been discussed in detail in chapter 5. In this section, however we will describe the structure and control flow of the system-learner dialogue.

While using TRAPS, a learner is given several related questions for each set of problems. The number of questions in a set of problems varies and so does the difficulty level of the questions.

An example of a system-learner dialogue during the TL session is as in Figure 6.1. 'T:' stands for questions or information from TRAPS, while 'L:' means the answer given by the learner. The questions are in multiple choice form. This is to make it easier for the learner to only enter a single character as their answer for each questions. After

---

<sup>1</sup> This is defined in section 3.6.

<sup>2</sup> Comparison is done manually by the teacher.

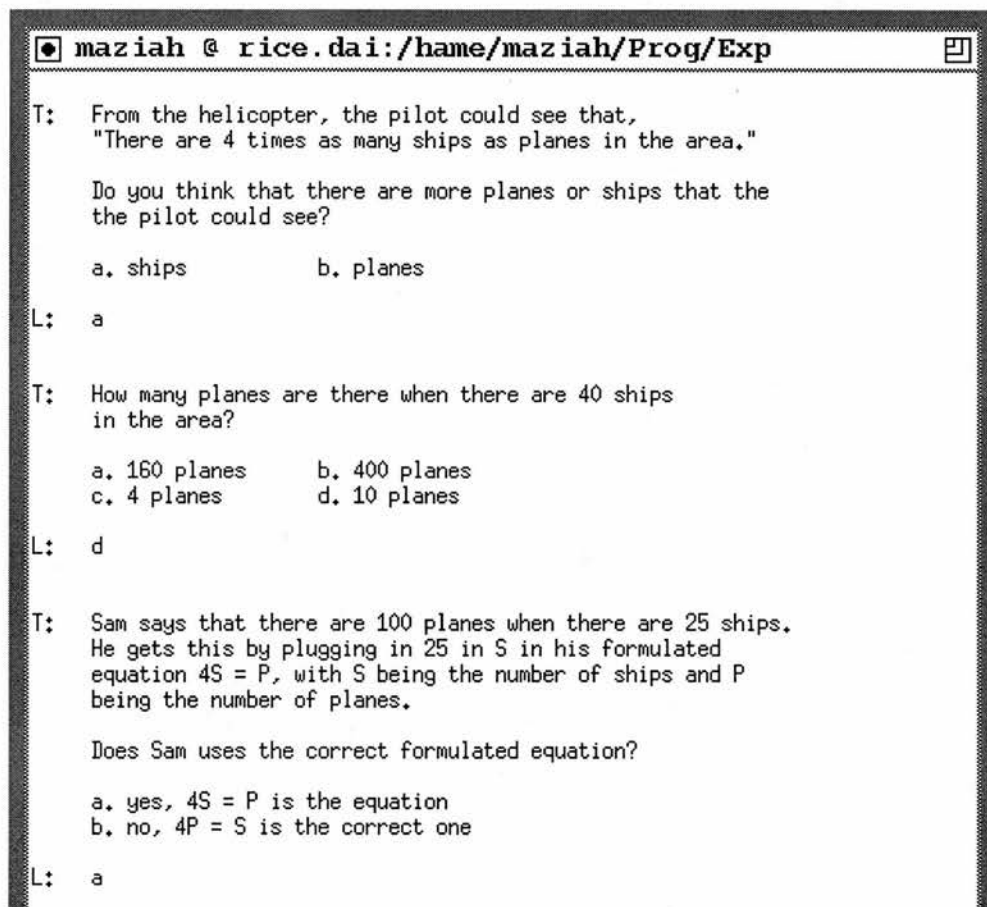


Figure 6.1: An example of a TRAPS-learner dialogue during the *teaching and learning* session.

each question has been asked, the learner is expected to type in his answer. Immediate feedback of whether the answer is correct or incorrect is not given at this stage. TRAPS proceeds with the next question and expects another answer from the learner. This is done until all the questions in the set of problem have been answered.

When the learner has answered all the questions in the problem set, TRAPS will give its feedback on the answers. TRAPS focuses on the occurrence or non occurrence of conflict between the learner's beliefs (his answers) and that of TRAPS's beliefs (its knowledge i.e the correct answers and the underlying concepts for each answers). Therefore, the feedback given will be based on the **existence or nonexistence of contradictory beliefs**. If no contradiction has been detected, this means the questions have been answered correctly (i.e the learner does not exhibit contradictory schemes in his reasoning for this particular problem set). In cases where a contradiction of beliefs has been detected (which in this example, it has), TRAPS will point this out to the learner. This is then followed by details about which of the two conflicting beliefs is correct ( $B_1$ ) and incorrect ( $B_2$ ), and which of the learner's answers relate to the incorrect belief.

A belief revision system keeps track of both the learner's and TRAPS' consistencies in beliefs during the online session of the teaching and learning session. This will be discussed further in several sections of this chapter.

### 6.3 Pre-defined question library

The questions posed in all of TRAPS' three sessions are pre-defined. As described in section 5.3, the questions are carefully designed and arranged using the *Cascaded Problem Set teaching architecture*. Under such a teaching architecture, the library is filled with questions in each cascade set. As defined by Schank, *Cascaded Problem Set teaching architecture* is to build a problem space whereby each problem relates to one another by the extra layer of complexity. For such a teaching architecture, according to Schank:

*“ ... it is necessary to build libraries of cascade and to determine the content-based connections that relate one problem with another. To do this,*



*a problem must be decomposed into its constituent parts. Each constituent would itself be a problem, and it too would have constituent parts. ”*

(Schank, 1990), p. 8

We adopt Schank’s idea of the *Cascaded Problem Set* teaching architecture by:

1. Designing our sets of word problems as the cascade sets. We refer to each of our sets of word problems as the *problem set* or *pset*.
2. Each pset has its constituent parts that consist of several questions. Every single question has its own underlying concept to test the learner’s understanding. Based on one or more **underlying concepts** and **presentation of statements** in the question<sup>3</sup> a value is assigned to indicate the level of difficulty of the particular question. This numerical value is referred to as the *question value* or *Qv*. Three categories of Qv are formed:

- simple, with the Qv of 1;
- moderate, with the Qv of 2; and
- difficult, with the Qv of 3.

We assess the correctness of assigning such values for each question, in one of the studies in our evaluation.<sup>4</sup>

3. Several of these psets are stored in a database called *Question library* or *Qlib*.

### 6.3.1 The Question library and the Question node

Psets are organised in the Qlib by layers of difficulty or complexity of questions in each of the psets. The difficulty level of a pset is measured by accumulating Qvs belonging to each questions in the pset. The accumulated numerical value is referred to as *Pset value*. These Pset values are the content-based connections that we use to relate one word problem set to another word problem set in Qlib. Qlib can be viewed as in Figure 6.2.

---

<sup>3</sup> This is as discussed in section 5.2.

<sup>4</sup> This is further discussed is in section 7.5.4.

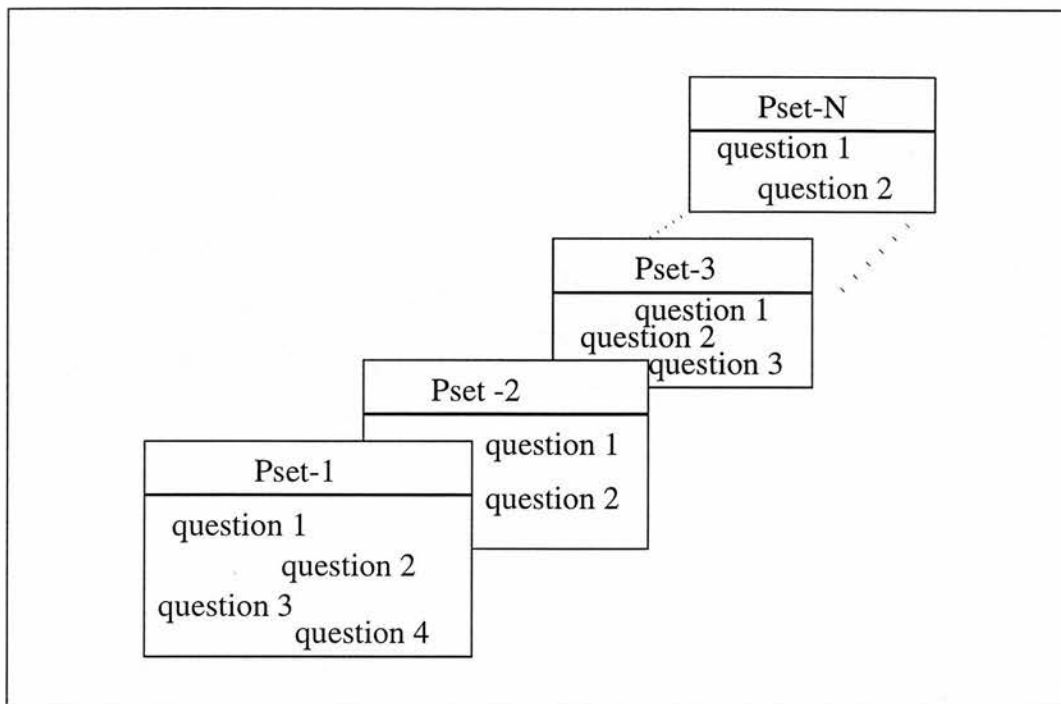


Figure 6.2: Each problem set is decomposed into several questions and arranged by their level of difficulty in a cascaded manner in the Question Library.

The sequence in which the word problems from Qlib are retrieved, depends on the learner's performance. TRAPS starts off with the simplest pset. If the learner's answers for the pset do not cause any contradiction of beliefs<sup>5</sup> and its detail implementation will be discussed in section 6.6, then the next most difficult question will be posed to the learner. This carries on until the most difficult pset. The reason for progressing from easier questions to more difficult ones is because this approach is more likely to elicit any entrenched misconception that may exist in the learner's reasoning process.

Information related to questions in the teaching and learning session will be registered in a database called *qnode*. This is done right after the learner has entered his answer for each question. Every single question in the teaching and learning session has one or more of its own qnodes. A question which has more than one underlying concept that TRAPS wants to test the learner will have more than one qnode.

The structure of each qnode is as:

<sup>5</sup> This is based on our theory of learner modelling discussed in section 5.5.

T: "There are 10 times as many people in China as there are in England".  
 Peter says that  $C = 10E$  is the correct equation formulated from the above statement.  
 Do you agree with his equation and what does C stands for in the equation?

- a. Yes, I agree with Peter's equation and C is the number of people in England
- b. Yes, I agree with Peter's equation and C stands for the people in China
- c. No, I disagree with Peter's equation and C is the number of people in England
- d. No, I disagree with Peter's equation and C stands for the people in China

*intended difference?*

*What message does this give?*

L: b

Figure 6.3: Several underlying concepts in this single question.

**qnode**(node number, problem set, question predicate, concept, question value, correct/incorrect, learner's answer).

The contents of a qnode is described as follows:

**Node number** acts as a counter for the particular problem set.

**Problem set** indicates the particular set of problem, eg. *pset(5)* is the *student-professor* problem.

**Question predicate** allows us to return to the question when we need to:

- display it again for the learner to correct his incorrect answer,
- display it in the learner's record of performance i.e the question he has difficulty with.

**Concept** holds the underlying concepts for a particular question. Some of the concepts have one value, while others may have more than one values.

As an example, question such as in Figure 6.3 will have three of its qnodes assigned — "accept the correct equation" or "reject the reversed equation" for its first qnode, "understanding of the concept of variable" for its second qnode and "reverse the meaning of the statement" for its third qnode.

**Question value** is the value of difficulty for the question. It could be 1, 2 or 3 representing simple, moderate or difficult, respectively.

**Correct/Incorrect** represents the correct or incorrect answer given by the learner for the particular question. It is represented as 1 or 0 i.e correct or incorrect, respectively.

**Learner's answer** holds the answer to the question that the learner believes.

A combination of any of these elements in qnode can act as a reference key for retrieving information from the qnode database.

## 6.4 Generating beliefs

In chapter 4, we have described and discussed TRAPS' belief revision system which is referred to as *BRILM* (**B**elief **R**evision **I**n **L**earner **M**odelling). As discussed in chapter 5, *BRILM* is the basis of our learner modelling technique. In this approach, beliefs are the main element that we utilise and maintain. How do we obtain the beliefs involved in our learner model? This section will discuss how we generate the beliefs involved, how we represent them in our program, and how we apply these beliefs in *BRILM*.

### 6.4.1 The representation of belief

The information processed as beliefs is represented as a propositional formula of (Berendt, 1992)'s object level logic. The belief sets are defined in terms of a meta level language, a sorted first-order language with equality. Both the object level and meta level language have been discussed in chapter 4.

Three types of beliefs are involved in *BRILM*:

1. TRAPS' own beliefs ( $B_T$ );
2. the learner's beliefs ( $B_L$ ); and
3. the *Learner Modelling* beliefs ( $B_{LM}$ ) i.e the implications made by TRAPS on the



```

/* Grammar */

sent --> start_sent, start_sent, qtt, r1, r2, qlt, obj1,
        r3, obj2, rest_sent, rest_sent, rest_sent.

start_sent --> [Ss], {ssent(Slist), findword(Slist,Ss)}.

qtt --> [Qt], {quant(Numlist), findword(Numlist,Qt)}.

qlt --> [Ql], {qual(Adjlist), findword(Adjlist,Ql)}.

obj1 --> [Obj1], {noun1(Nlist), findword(Nlist,Obj1)}.

obj2 --> [Obj2], {noun2(Nlist), findword(Nlist,Obj2)}.

rest_sent --> [Rs], {rsent(Rlist), findword(Rlist,Rs)}.

r1 --> [Rel], {relt(Relist), findword(Relist,Rel)}.

r2 --> [Rel], {relt(Relist), findword(Relist,Rel)}.

r3 --> [Rel], {relt(Relist), findword(Relist,Rel)}.

```

Note: findword/2 selects a random element from a list.

---

Figure 6.5: The grammar in TRAPS' DCG.

word problems. We use this simple DCG to extract the relevant keywords to generate specific beliefs that belongs to TRAPS,  $B_T$ . This is implemented for the ten sets of word problems.

TRAPS' own beliefs,  $B_T$  which are generated for each word problems are listed as follows. In order to make the discussion clearer, we will give example for each beliefs based on one of TRAPS' questions:

*From the helicopter, the pilot could see that , "there are 4 times as many ships as planes in the area". Use S for the number of ships and P for the number of planes when translating the statement into an equation.*

Statements quoted in italic in the list below are the beliefs generated for  $B_T$  for this ship-plane word problem.

- Correct translated equation.

```

/* Vocabulary */
ssent([there, are, is]).

quant([1, 2, 3, 4, 5, 6, 7, 8, 9, 10]).

qual([many, much, less, little]).

noun1([children, students, professors, teachers, planes, ships,
       cows, sheep, vinegar, oil, chinese, english,
       tourist_in_edinburgh, tourist_in_london]).

noun2([students, professors, teachers, adults, planes, ships,
       cows, sheep, vinegar, oil, chinese, english,
       tourist_in_edinburgh, tourist_in_london]).

rsent([in, at, during, this, the, university, school, area, field,
       farm, kindergarten, playground, food, salad_dressing,
       countries, festival_period]).

relt([times, as]).

```

---

Figure 6.6: The vocabulary in TRAPS' DCG.

*"The ship-plane equation is  $S = 4P$ "*

- The reversed equation.

*"The ship-plane reversed equation is  $4S = P$ "*

- The qualitative value.

*"There are more ships than planes"*

- The opposite qualitative value.

*"There are more planes than ships"*

- The concept of variable as unspecified quantity.

*"S stands for the number of ships"*

*"P stands for the number of planes"*

- The concept of coefficient as a number to be operated on by one of the variables given.



*"P must be operated on by multiplication of 4 to produce a number that is the same as S"*

- The concept of equal sign as an equivalence operator for both groups that are involved in the given word problem.

*Operation on P and the coefficient 4 is to equalise the number of P and S.*

These beliefs are generated as TRAPS' own correct beliefs on the particular problem. They are used for comparison with the learner's beliefs. As discussed in chapter 5, sections 5.4 and 5.5, TRAPS' approach to diagnosing the existence of contradictory schemes in the learner's reasoning is by detecting any occurrence of conflicting beliefs — **between TRAPS' and the learner's beliefs and also within the learner's own beliefs.**

#### 6.4.3 The learner's answers

The learner's beliefs are generated online when he answers the questions. Each answer given is translated to an appropriate belief. The translation of these answers is based on a simple *if-then-else* checking. TRAPS matches each answer to a proper belief i.e *if the learner's answer is a, then he believes X, else if the learner's answer is b, then he believes Y*, and so on depending on how many choices of answers are given to the learner. The belief either X, Y or others will be added to BRILM as shown in Figure 6.8. For example, referring to TRAPS' question and the learner's answer to the question in Figure 6.7, the learner's belief that is generated and added to BRILM is *"the pilot could see more ships than planes"*.

#### 6.4.4 Implication of the learner's answers

The Learner Modelling beliefs,  $B_{LM}$  are TRAPS' implications of the learner's beliefs. Using the learner's belief obtained from the answer in Figure 6.7, the  $B_{LM}$  being generated is *the learner believes that there are more ships than planes*. This is regarded as the learner's qualitative understanding. This can be seen in Figure 6.9.

T: From the helicopter, the pilot could see that,

"There are 4 times as many ships as planes in the area."

Do you think that there are more planes or ships that the pilot could see?

a. ships                      b. planes

L: a

Figure 6.7: A question posed to the learner. The answer determines the input that will be generated as the learner's belief for this question. The belief to be generated from either of these answers is shown in the next figure.

```
cqqlt6(A):- (A == a),
             Sev is 0.9,
             add_assumption(answers(student, 'the pilot could see
                                     more ships than planes'), Sev),
             :
             :

cqqlt6(A):- (A == b),
             Sev is 0.9,
             add_assumption(answers(student, 'the pilot could see
                                     more planes than ships'), Sev),
             :
             :
```

Figure 6.8: Based on the learner's answer, an appropriate learner's belief will be added to BRILM.

## 6.5 Adding beliefs to BRILM

The generated beliefs are added into a belief set<sup>7</sup> in BRILM. We add each belief together with its entrenchment value. The entrenchment value is classified as:

1. *Sev* - Learner's (student's) entrenched value;
2. *Tev* - TRAPS' entrenched value;
3. *Tov* - TRAPS' observation's (meta level) entrenched value.

*for what purpose?*

<sup>7</sup> A set that is to contain all the beliefs

```
imply_answers(pset(6)):-
```

```
Tev is 1,
add_assumption('the pilot could see more ships than
                planes' -> 'there are more ships than
                planes', Tev),

add_assumption('the pilot could see more planes than
                ships' -> 'there are more planes than
                ships', Tev),
:
:
```

*What are the conditions?*

Figure 6.9: Each of the learner's answers has its own implication. Only existing beliefs (that have been added as the learner's belief) will have their implication belief generated and added as the Learner Model's belief.

Each of these generated beliefs has its own entrenchment value fixed at the beginning of the session for each new pset. The learner's belief is assigned a numerical value lower than that of TRAPS. We regard their beliefs as less reliable than that of TRAPS' own beliefs. We choose 0.9 to be the degree of reliability for *Sev*. TRAPS own beliefs are assumed to be correct and constructive<sup>8</sup>. In Berendt's meta language, constructive tautologies have a degree of 1. Hence, we assigned the value of 1 for *Tev*. As for TRAPS's observation, *Tov* we assigned it to be 0.95, as it may not be a definite correct observation.

*though they are not tautologies.*

The beliefs and their entrenchment values are added into the belief set as follows:

*learner\_belief:- Sev is 0.9, add\_assumption(answers(learner,X) → X, Sev).*

*traps\_belief:- Tev is 1, add\_assumption(says(TRAPS,X) → X, Tev).*

*traps\_implication:- Tov is 0.95, add\_assumption(X → Y, Tov).*

The following is a brief description of the underlying process of the example in Figure 6.11, and Figure 6.12. Figure 6.11 is an example of eliciting a learner's entrenched belief. Here, the learner translated to the correct equation. Through the cascaded problem set teaching architecture we adopt, the third question is posed to determine whether the learner understands the expression he had formulated. In other words,

<sup>8</sup> This has been mentioned in section 4.8.

```

% Implication of the reversed equation (if it is being
% formulated) for problem set 6

imply_re(pset(6)):-

% Retrieve earlier generated information from the data base -
% the quantity, object1, object2, variable associated to
% object1 and object2

vqtt(Q), vobj1(Ob1), vobj2(Ob2),
ov(Ob1,Xo1), ov(Ob2,Xo2),

% Form the belief on qualitative value that will be
% implied if a reversed equation has been formulated

Qtycomre = ['there are more ', Ob1,' than ', Ob2],
Qtycomre1 = ['there are more ', Ob2,' than ', Ob1],

% Initialize and store the proposition

retractall(bs_qty_comparisonre(_)),
retractall(bs_qty_comparisonre1(_)),
assertz(bs_qty_comparisonre(Qtycomre)),
assertz(bs_qty_comparisonre1(Qtycomre1)),

% Change the proposition from a list structure to an atom

list_to_atom(Qtycomre, AQtycomre),
list_to_atom(Qtycomre1, RQtycom),

% If a reversed equation exist in the belief set,
% an implication of contradiction in ''qualitative
% value'' will be triggered

Tev is 1,
add_assumption('a reversal of the correct equation for
                the ship-plane problem' -> RQtycom, Tev),
add_assumption( RQtycom -> neg AQtycomre, Tev).

```

---

Figure 6.10: Implication of formulating the reversed equation - causing a contradiction on the "qualitative value" belief proposition.

T: Jean wants to prepare a home-made salad dressing.  
According to the recipe, she has to mix 3 times  
as much oil as vinegar.  
Does Jean has to put more oil or vinegar?

- a. vinegar      b. oil

L: b

T: Now, try formulate an algebraic equation of the  
statement "There is 3 times as much oil as vinegar  
in the salad dressing", using the letter L for the  
amount of oil and V for the amount of vinegar.

- a.  $3L = V$       b.  $3V = L$

L: b

T: You've formed the correct equation, this time.

So, according to the equation  $3V = L$ , if Jean puts  
a cup of oil, she will have to mix it with 3 cups  
of vinegar.

Do you agree with that? Yes/No (y/n) ?

L: y

---

Figure 6.11: A sample of the diagnosing session with TRAPS.

>< >< >< CONFLICT DETECTED >< >< ><

Aha ... there's a conflict here. You seems to believe  
that Jean has to mix more vinegar than oil  
as well as its opposite,  
that Jean has to mix more oil than vinegar  
at the same time.

---

Figure 6.12: Level 1 conflict has been detected by the Belief System and TRAPS is being notified.

TRAPS is testing if the learner's misconceptions (which were detected previously) on *the concept of variable, equation and equal sign* resurface. His answer in this case indicates that even though he had formulated the correct equation, he reverses the original meaning of the problem statement. The learner's misconceptions on the *concept of variable, coefficient and equal sign* still persist in his mind.

An illustration of the underlying process is as follows.

$B_L$  - Learner's beliefs, with  $Sev$  as the entrenchment value

$B_L1$ : Jean has to mix more oil than vinegar

$B_L2$ : the equation is  $3V = L$

$B_L3$ :  $3V = L$  means that for a cup of oil, mix 3 cups of vinegar

$B_T$  - TRAPS' own beliefs, with  $Tev$  as the entrenchment value

$B_T1$ : mix more oil than vinegar

$B_T2$ : the equation is  $3V = L$

$B_T3$ :  $3V = L$  means that for a cup of oil, mix  $1/3$  cup of vinegar

$LM$  - The Learner Modelling, i.e TRAPS' implications of the learner's beliefs, with  $Tov$  as the entrenchment value

$LM1$ :  $B_L1 \rightarrow$  mix more oil than vinegar

$LM2$ :  $B_L2 \rightarrow$  the amount of oil is more than vinegar

$LM3$ :  $LM2 \rightarrow$  mix more oil than vinegar

$LM4$ :  $B_L3 \rightarrow$  mix more vinegar than oil

$LM5$ :  $LM4 \rightarrow$  negate(mix more oil than vinegar)

Hence, a contradiction between "*mix more oil than vinegar*" and "*negate(mix more oil than vinegar)*" occurs as shown in Figure 6.12. For the purpose of the interface in TRAPS, "*negate(mix more oil than vinegar)*" is displayed as *mix more vinegar than*

oil.

BRILM resolves the conflict by leaving “*mix more oil than vinegar*” and its justifications in the belief set, and dropping “*mix more vinegar than oil*” and its justifications from the belief set. TRAPS’ teaching component in the system will be notified by BRILM, and all the justifications involved will be *handed over* to be used as part of the explanation<sup>9</sup> to the learner.

## 6.6 Handling contradiction of beliefs

Contradiction is resolved in BRILM by the preference ordering of beliefs which is based on Gärdenfors’ notion of *epistemic entrenchment* (Gärdenfors, 1988). Here, the two beliefs involved in a contradiction are revised based on their *strength of belief*, the beliefs’ entrenchment values.

In the triangle conflict or Level-1 conflict, the competition of beliefs is between *Tev* and *Sev*. *Tev* is always greater than *Sev* since ideally beliefs (knowledge) of TRAPS about the domain are always assumed to be correct. Hence, the initial belief (that leads to a contradiction) with *Tev* will be regarded as more valid and will survive in the competition, while the one with *Sev* will be discarded. This revision process occurs in the BR system. BR will then notify TRAPS that the disqualified belief and its justifications are the ones that the learner has to withdraw from his reasoning process.

In the observation conflict or Level-2 conflict, however, the competition of the beliefs now is between two *Tovs* where both have the same entrenchment value. Berendt’s BR system applies Gärdenfors’ *conservative* way of choosing the belief that is to be kept when resolving a conflict. In this approach, the earlier belief is considered to be more reliable compared with the newer ones. Unfortunately, this is not the case during a teaching session. An earlier observation should not necessarily be assumed to be more valid than a recent one. In BRILM, we evaluate the abstract properties of the questions that the learner has been asked and relate these entities to *Tov*. The ones with stronger justifications will not have their entrenchment value decremented and thus survive in the belief set.

---

<sup>9</sup> This will be discussed in section 6.7.1.

*comment as before*



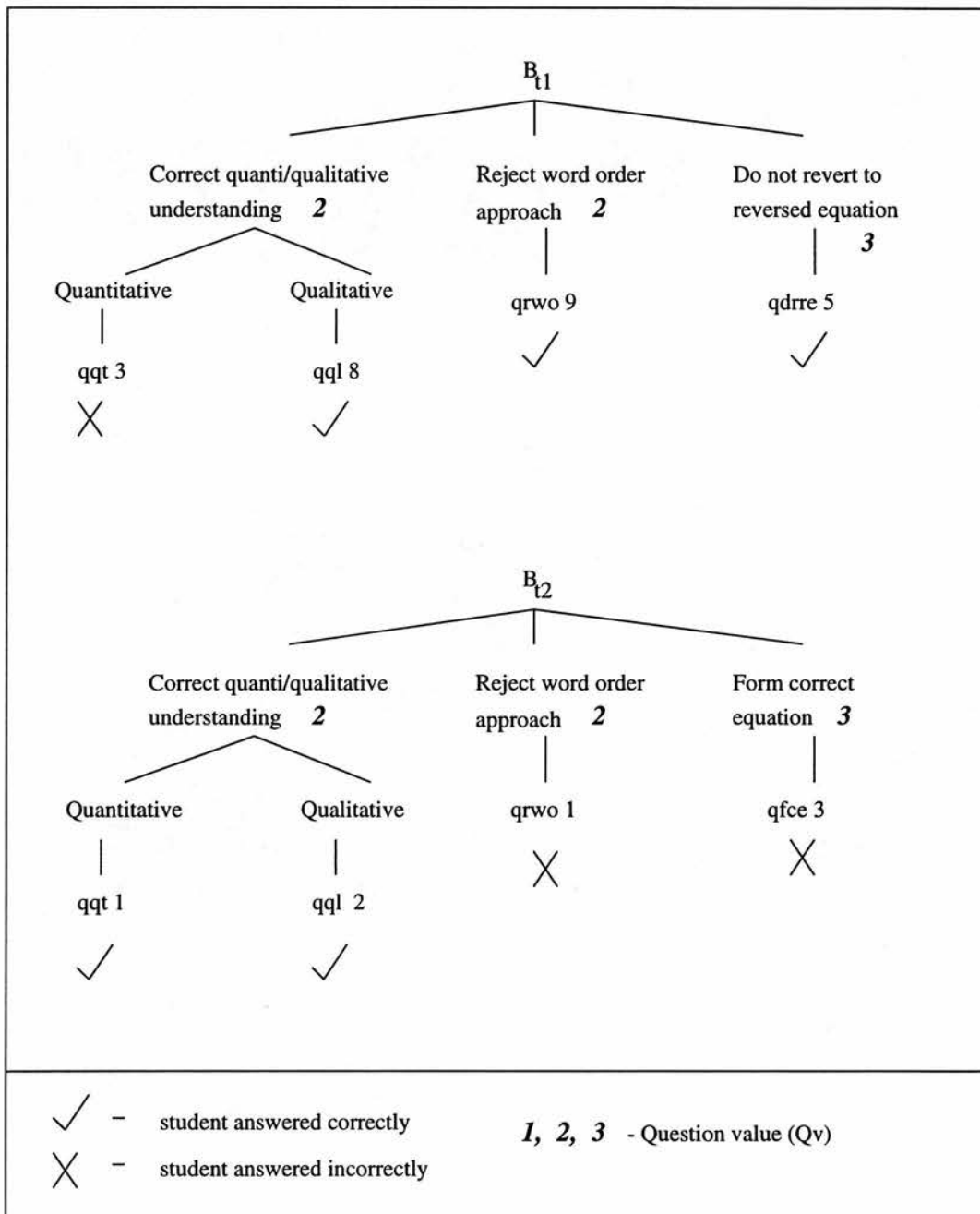


Figure 6.13: Evaluating the question values for conflict between “the learner is capable of translating relational algebra word problems” and “the learner is not capable of translating relational algebra word problems”.

When competition between two *Tovs* occurs, TRAPS will refer to the related justification from BR's backward chaining of reasoning which ends up with the beliefs identifying the learner's answer to the related question. TRAPS will access the questions involved and examine whether the learner answers each of these questions correctly or incorrectly. Only those answered correctly will be taken into account i.e their *Qv* will be added up to produce the total value as *Tqv1*. The same evaluation is done for the other competing belief which will result in *Tqv2*. These two values (*Tqv1* and *Tqv2*) are compared. The one weighted heavier will have its related *Tov* incremented. *was made?*

An example as shown in Figure 6.13 is described as follows. A contradiction of TRAPS believing that (*the learner is capable of translating the relational algebra word problem*), referred to as  $B_{t1}$  and (*the learner is incapable of translating the relational algebra word problem*), referred to as  $B_{t2}$  occurs. The questions involved in producing  $B_{t1}$  are qqt3 (question on quantitative understanding number 3), qql8 (question on qualitative understanding number 8), qrwo9 (question on rejecting word order approach number 9), and qdrre5 (question on do not revert to reversed equation number 5). The learner answered correctly for qql8, qrwo9 and qdrre5. Only those correctly answered will have their *Qv* counted. In this case, the *correct quantitative-qualitative understanding* property is not counted since only qql8 is correctly answered but not qqt3. Hence,  $B_{t1}$  has its *Tqv1* with the value of 5. *Tqv2* for  $B_{t2}$  is only 2. Since *Tqv1* is greater than *Tqv2*,  $B_{t1}$  will be the one favoured by the BR system to survive in the contradiction. However, TRAPS having gone through the evaluation of *Qv* will take note that the learner is "*able to translate the relational algebra word problem*" but the learner still needs more practice on questions that he had made mistakes on.

## 6.7 Explanation generator

The justifications of contradicting beliefs are used for explanation during the remediation session. (Kass & Finin, 1988) highlighted familiarity as a characteristic of understandable explanations. Hence, using the learner's own beliefs in the explanation provides information that he is familiar with. This could help his understanding in remediating his own misconceptions. Among the issues that (Murray et al., 1990)'s concern regarding deep-seated misconceptions is:

*“Careful sequencing and explanation of new information, without using knowledge of the learner’s prior beliefs, is probably not sufficient to lead to understanding. Existing misconceptions must be directly addressed ...”*

(Murray et al., 1990), p. 82

We adopt (Murray et al., 1990)’s idea due to the fact that misconceptions related to the reversal error are usually deeply entrenched<sup>10</sup> in the learner’s reasoning process. We integrate both (Murray et al., 1990)’s and (Kass & Finin, 1988)’s ideas in TRAPS’ explanation.

As earlier discussion on TRAPS’ explanation in section 5.5.2, TRAPS produces two types of explanation — the *Learner-TRAPS-beliefs explanation* and the *Partial Canned Text explanation*.

#### 6.7.1 The Learner-TRAPS-beliefs explanation

Explanation which contains the learner’s and TRAPS’ specific beliefs which are involved when the contradiction occurs, are referred to as the **Learner and TRAPS Beliefs explanation** (*LTB-explanation*). These beliefs are retrieved by tracing back from the conflict belief to their *assumption*<sup>11</sup> bases. An example of an online session of questions (on the *playground problem*) and conflict detected followed by an LTB-explanation is as in Figure 6.14.

#### 6.7.2 The Partial Canned Text explanation

However, the LTB-explanation focuses on the logic flow of the beliefs and justifications involved in the conflict detected. It does not explain the overall approach of translating the relational statement into equation. Hence, another type of explanation focusing on the strategies of translation is generated as a supplement to assist the learners. This other type of explanation has contents that describe the translation strategy which the learner had applied and the correct translation approach that TRAPS has. This type

---

<sup>10</sup> This is discussed in section 3.4.1.

<sup>11</sup> A self-evident belief in BRILM.

● maziah @ rice.dai:/am/rock/disk/home/rock10/phds.9

T: 'There are 7 times as many children as adults at the playground.'  
The number of adults are \_\_\_\_ the number of children at the  
playground

a. more than    b. less than    c. the same as

L: b

T: To make the number of children (C) more than the number of  
adults (A), we should multiply 7 to C, so that C will be more  
than A. Yes/No ?

L: y

××× CONFLICT DETECTED ×××

Aha ... there's a conflict here.

You seem to believe that  
    'there are more children than adults'

AND ALSO

    'there are more adults than children'  
at the same time.

The belief - 'there are more adults than children' arises from:

1. Your answer - 'multiply 7 to C, so that C will be more' which leads to  
    'the coefficient 7 is being associated with C to make C as the  
    larger group'

which leads to

    'a reversal of the correct equation'

which then leads to

    'there are more adults than children'.

The belief - 'there are more children than adults' arises from:

1. Your answer - 'the number of adults are less than the number of children'  
which leads to

    'there are more children than adults'

2. According to TAPS - 'there are more children than adults'.

Now, do you want to:

a. get further explanation    or    b. fix the answer

L: a

Figure 6.14: A triangle conflict had been diagnosed. The LTB-explanation is shown here as the remediation session for the learner.

```

% Retrieve the database to fill in the partial canned text

template_filler(Q, Obj1, Obj2, Var1, Var2, CE) :-

vqtt(Q), vobj1(Obj1), vobj2(Obj2),
ov(Obj1,Var1), ov(Obj2,Var2), creqt(CE).

taps_explain(statcomp):-

template_filler(Q, Obj1, Obj2, Var1, Var2, CE),
taps_says,
write('The letters '), write(Var1), write(' and '),
write(Var2), write(' in the equation '),
write(Q), write(Var1), write(' = '), write(Var2),
nl, tab(5),
write('from the problem statement do not stand
for the'), nl, tab(5),
write('physical object. '), nl, tab(5),
write('That is '), write(Var1),
write(' does not represent the '),
write(Obj1), write(' and '),
nl, tab(5), write(Var2),
write(' does not stand for the '),
write(Obj2), write('.'),
nl, nl, tab(5), write(Var1), write(' and '),
write(Var2),
write(' are variables that represent some number.'),
:
:

```

---

Figure 6.15: Generating the Partial-Canned Text explanation for *static-comparison approach*.

of explanation is referred to as the **Partial Canned-Text** explanation or the (*PCT-explanation*) since it is generated from partial canned-text catering for the specific problem sets. Figure 6.15 shows how the *template filler* is retrieved from the database and filled into the templates between the canned text. The explanation produced by generating this code is in Figure 6.16.

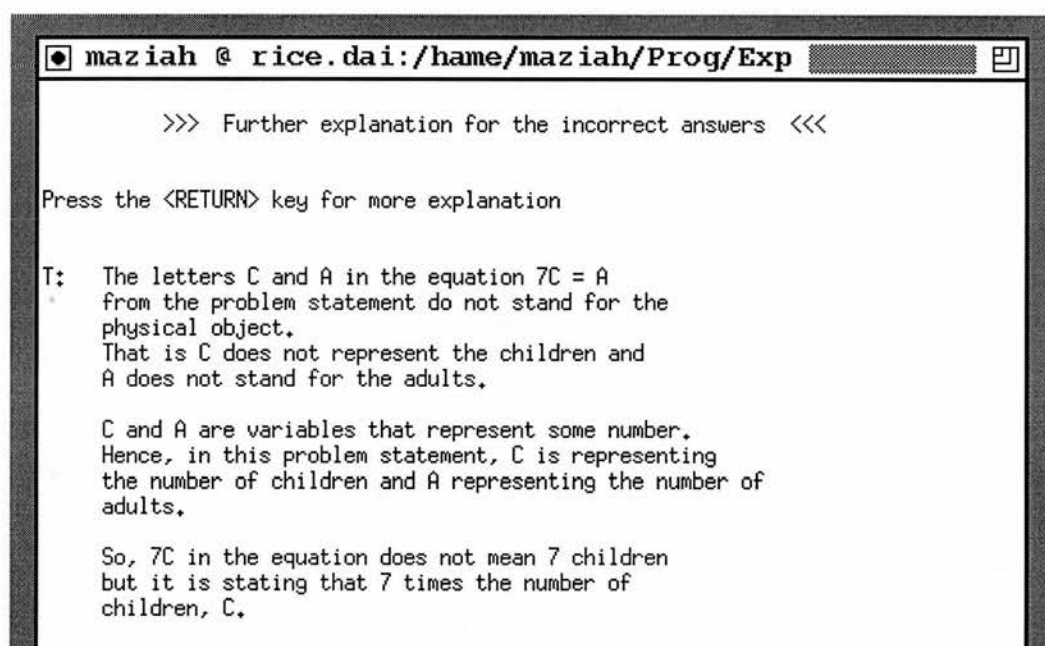


Figure 6.16: A Partial Canned Text explanation for *static-comparison approach* incorrectly applied to the *playground problem*.

## 6.8 The learner's history record generator

Information related to questions in the teaching and learning session is stored in the *qn-ode* database as described in section 6.4.1. Data in qnodes provide TRAPS information that can be produced as the learner's performance record.

After the explanation session, in a triangle or Level-1 conflict, the data we retrieve from the qnode are the learner's answers, the correct/incorrect value and the question predicate. Our particular interest is in the incorrectly answered questions. At this level of conflict, the incorrect answer is shown to the learner. We then return to the question predicate to display the question again. This allows the learner to fix his incorrect answer, after explanation has been given to him. An example of this is in Figure 6.17.

When evaluating a learner's performance, there are times when a teacher at first believes that the learner has mastered the subject material (e.g concepts that are needed when formulating an algebra equation from a relational sentence). But from some new evidence (after several exercises), the teacher then discovers that the learner has not actually mastered the material yet. We consider this as another stage of conflict of beliefs i.e within the teacher's own beliefs.

In TRAPS, such a situation can be traced. We show in this section a sample of TRAPS' observation on two different problem sets i.e problem set 1 and 6. There is a contradiction of TRAPS' beliefs from this observation. The beliefs involved are shown. An example of such a situation can be viewed in Figure 6.18. As can be seen from this figure, such an explanation for Level-2 or meta level conflict displays the learner's record of performance. This record is provided only for the teacher and not the learner. Based on our evaluation among teachers<sup>12</sup>, such a record is useful for the teacher to know the learner's difficulties or problematic concepts.

---

<sup>12</sup> This will be discussed in detail in chapter 7.



```
maziah @ rice.dai:/hame/maziah/FIG

X X X CONTRADICTION DETECTED X X X

T: Aha ... there's a conflict here. You seem to believe:
    there are more teachers than children
AND ALSO
    there are more children than teachers
at the same time.

The belief
    there are more children than teachers
and its justifications are correct

but its opposite belief,
    there are more teachers than children
and the reason it arises, due to

Your answers in this problem set -

    1. C and T stand for children and teachers
    2. can translate by following the order of the keywords in the sentence
    3. the equation for the kindergarten problem is  $5C = T$ 

need to be fixed.

    Now, would you like to

    a. get more explanation on your incorrect answer(s)
    b. fix your answer(s)

L: b
    >>> Fixing the incorrect answers <<<

T: Okay ... Let's get back to the question(s) where you have
    to fix the answer(s).

T: In your translated equation, C and T each (respectively)
    stands for :

    a. children and teachers
    b. a child and a teacher
    c. the number of children and the number of teachers

L: a

T: Oops... you got it wrong again!
    Each variable does not refer to the physical object.
    It stands for a number, indicating the quantity of the object.
    C and T stand for the number of children and the number of teachers.

    Press <RETURN> to continue ...

T: We can translate this statement by just following
    the order of the keywords (5, children, teachers) in the
    sentence given.
    Yes/No (y/n)?
```

Figure 6.17: Displaying to the learner his incorrect answers. If he chooses to fix his answer, the related question will be posed.

>>> Conflict in observation between problem sets <<<

A contradiction between  
``the learner is not able to translate RAWP''  
and  
``the learner is capable to translate RAWP''  
has occurred.

The belief - the learner is not able to translate  
RAWP arises from:

1. According to TAPS -  
learner scores lower than average and there  
occurs a conflict of beliefs in Problem Set 1

which leads to

the learner is not able to translate RAWP

=====

The belief - learner is able to translate RAWP  
arises from:

1. According to TAPS -  
learner scores higher than average and  
no conflict of beliefs in Problem Set 6

which leads to

the learner is able to translate RAWP

=====

Looking back at the metalevel observation,

The belief - learner is able to translate RAWP  
and its underlying beliefs are more justified.

However, the opposite belief -  
"learner is not able to translate RAWP"  
and its causes is due to the learner's incorrect answers in:

Problem Set 1, Question 5, misconception on coefficient.  
Learner's answer was:  
'multiply 5 to C to make it the larger group'

Problem Set 1 - Question 4, misconception on variables  
Learner's answers was:  
'C and T stands for children and teachers'

seems to be a bit of a problem to the learner.

Figure 6.18: A meta-level observation conflict and part of its explanation. This information is recorded in a separate file for the teacher's reference. Note: RAWP stands for Relational Algebra Word Problems.

## 6.9 Conclusion

This chapter has discussed each component of TRAPS. The discussion covers the structure, implementation, usage and examples for each component. TRAPS is implemented in SICStus Prolog. This is due to the fact that BRILM, being our major component in TRAPS is based on Berendt's BRS<sup>13</sup>. BRS is conveniently implemented in SICStus Prolog since BRS deals with manipulation of logical expression that represent beliefs in it. SICStus Prolog which is available in our department supports the needs of BRS, BRILM and TRAPS as the whole system in our research.

In the next chapter, we describe and discuss the experiment conducted to evaluate the implemented prototype system, TRAPS. The observations and results are also presented.

---

<sup>13</sup> As discussed in chapter 4.

## Chapter 7

# Evaluation

In this chapter we discuss the evaluation of TRAPS as a prototype system. Categories of evaluation in ITS in general are described at the beginning of this chapter. This is then followed by a short description of our formative-internal evaluation and formative-external evaluation. The stages, subjects, materials and methodologies involved in the latter evaluation are discussed. The observation and results of these stages of evaluation are also reported. Finally a summary of TRAPS's evaluation is discussed.

### 7.1 Categories of evaluation

Two stages of evaluation are considered when evaluating an *Intelligent Tutoring System* (ITS) — **formative** and **summative** evaluation (Mark & Greer, 1993).

**Formative evaluation** is the evaluation made while the system is in its development process. The evaluation is continually being assessed to find out whether the development of the system meets the requirements set at the design stage. In contrast, **summative evaluation** is the evaluation of a completed system. The purpose of this evaluation is to establish the educational impact of the system.

Another description of evaluation is in terms of **internal** and **external** evaluation (Littman & Soloway, 1988).

**Internal evaluation** is carried out to assess the relationship between the component in the system's architecture and its performance. The evaluation is done extensively to test how each individual module of the system operates and how each module performs

in conjunction with other modules. The goal of an internal evaluation is to ensure that the program works as expected.

**External evaluation** is meant to assess the user's experience of the system and its impact on the user's knowledge of the domain in the system. External evaluation can be carried out as either formative or summative evaluation (Littman & Soloway, 1988), depending on the completion stage of the system. However, it is seen to be important at the formative evaluation stage in guiding the development of the system.

TRAPS is very much a prototype system, exploring the idea of applying a belief revision system in its learner modelling for a domain where contradictory schemes<sup>1</sup> exist. Our research at this stage is not aiming to develop a fully sound and effective pedagogical tutoring system. In other words, TRAPS is not a complete ITS but rather a system embodying a learner modelling technique that is being investigated.

Hence, TRAPS' evaluation is limited to formative evaluation. Within our formative evaluation, we carry out the internal and external evaluations which we refer to as the **formative-internal evaluation** and **formative-external evaluation**.

## 7.2 Formative-internal evaluation

It is necessary to consider evaluating the individual components of TRAPS. This is to assess to what extent their behaviour correlates with the aims at the design stage. We tested the performance of the system as designed, for each sessions in TRAPS, i.e the Pre-test, Teaching & Learning session and Post-test.

1. Pre-test - generates the learner's score, detects his misconceptions and the strategy he used.
2. Teaching & Learning (TL session) - detects contradictions from series of answers given by the user and implication of beliefs from the answers. Formulates explanation that heavily depend on information obtained when the contradiction is being resolved. Mainly, in this session the integration of the belief revision system plays a major role.

---

<sup>1</sup> This was defined in section 3.6

3. Post-test - generates the learner's score, detects his misconceptions and the strategy being applied, A simple report indicating the learner's score, misconceptions and strategy used in both the pre-test and post-test is also produced.

As described above, the pre and Post-tests were to function similarly. They share the same programming code. The differences were that they were executed at a different time and on different set of questions, and a report of the learner's score, misconceptions and strategy used in both the pre-test and post-test is produced after the post-test. This report is produced and compiled in a file that can be retrieved by teachers. It is only for the teachers and not meant for the learners and hence the learners are not notified about this report.

### **7.2.1 Methods and outcome of this evaluation**

TRAPS' formative-internal evaluation was conducted by systematically testing simulated examples of each misconceptions.

Generating the learner's score could be performed easily and during our testing, a mistake was detected when it missed incrementing a few questions. Detecting the learners' misconceptions was based on the underlying concept of the pre-designed questions. Corrections were needed when we discovered a different concept and misconception was mistakenly assigned to a few questions. The codes to identify the strategies used by the learners did not need to be changed since we have managed to assign appropriately the type of questions and underlying concepts and strategy of solving the question.

Extensive testing was made in the teaching and learning sessions. In particular when integrating the Belief revision system with TRAPS-learner interface. Changes made to the belief revision system have mainly been discussed in chapter 4.

## **7.3 Formative-external evaluation**

It was stated earlier that our formative-internal evaluation of TRAPS involved testing the program's ability to diagnose and remediate (as designed) contradictions of beliefs in the domain of relational algebra word problem.

The other portion of our evaluation is the formative-external evaluation. Our formative-external evaluation was conducted on students and teachers. This external evaluation was carried out to evaluate TRAPS' potential educational impact, at its prototype stage.

Our assessment consisted of a small scale formative evaluation based on a number of subjects. We ran a small scale experiment for the following reasons:

1. As previously mentioned, TRAPS is not a complete ITS but rather embodies a learner modelling technique that is being investigated. If we were to experiment with TRAPS in a real classroom environment, this could possibly lead to learners committing unexpected errors, which TRAPS would not be able to handle.
2. We limit the misconceptions to the most common and frequent ones in relational algebra word problems. Hence, varieties of misconceptions and mastery levels of too many learners may swamp the evaluation to other experiment variables. Such a situation could complicate our testing of the effectiveness of TRAPS learner modelling techniques within TRAPS' defined scope.
3. TRAPS runs on work-stations rather than on personal computers. The volunteers have to come over to our department to engage themselves with TRAPS' online session. This is not convenient for large numbers of volunteers.

TRAPS' formative-external evaluation addresses the following questions:

1. Does the learner have a positive and useful experience when using TRAPS?
2. Do the explanations of misconceptions leading to a reversal error help the learner to avoid repeating the error again?
3. Do the beliefs derived and interpreted by TRAPS differ from those of the teachers' derivation and interpretation?
4. Is the learner modelling technique in TRAPS useful for teachers in assisting learners with entrenched misconceptions in the domain of algebra word problems?



### 7.3.1 Forms of formative-external evaluation

We conducted two forms of formative-external evaluation, one with learners and the other with teachers.

The **first formative-external evaluation** was more concerned with the learner's experience of the system and whether it helps the learner to avoid committing the reversal error, addressing the first two questions above. We break our first external evaluation into two parts:

**Study 1** - Involved two learners. It served mainly to remove remaining bugs in the program and improve it sufficiently for further assessment of TRAPS by the next group of learners, in Study 2. The aim of the first stage evaluation is to identify users' problems and concerns after using TRAPS.

**Study 2** - Involved five learners. It was conducted after TRAPS had been debugged, its interface improved and English wordings altered through comments obtained from Study 1. Its aim is to evaluate TRAPS' effectiveness in diagnosing and remediating learners who commit reversal error. Study 2 required more of the learners' time compared to that of Study 1. It consisted of more materials and thorough observation also. Study 2 formative evaluation revealed some remaining problems and limitation of TRAPS. It also reinforced similar findings from the Study 1 and also provided some new unexpected outcomes.

The **second formative-external evaluation** was carried out to get feedback from teachers in assessing TRAPS' learner modelling technique from the perspective of educators. We refer this second formative-external evaluation as **Study 3**. In Study 3, four teachers were involved.

## 7.4 First formative-external evaluation - Study 1 and 2

This section describes the subjects, materials, and methodology involved in the first formative-external evaluation. Outcomes of the evaluation are also discussed at the end of this section.

### 7.4.1 Subjects

Seven learners were involved in this evaluation. Although these learners vary in terms of age and academic background, all of them have good a command of English. The latter criteria was considered important to avoid the language barriers as a factor that might influence the learners' performance in using TRAPS. The subject were two secondary school students and five undergraduates. Each of them had a secondary school level mathematics background. Their level of performance in mathematics was not known before the evaluation.

Study 1 was carried out with two of the undergraduate students. Two secondary school and three undergraduate students were involved in Study 2 of this evaluation.

### 7.4.2 Material

In both Study 1 and Study 2, all the learners were to answer questions asked by the system. The questions are from three different sessions with TRAPS, in the following sequence — pre-test, teaching & learning session and post-test.

All questions were of a multiple-choice type. In the pre-test and post-test sessions, no feedback was given to the learners. Questions in the pre-test and post-test are isomorphic to each other. *what does this mean, anyway?*

*read at least 1 page before*

As for the teaching & learning session, questions from several problem-sets were displayed for the learners to answer. The problem-sets were retrieved from the problem-set library. A complete set of questions from a problem-set were posed once. If the learner made a reversal error, the related questions (that leads to the reversal error) from the problem-set were redisplayed for the learner to fix.

Upon finishing the online sessions, a short structured interview was carried out with the learners. Questions asked addressed the following issues:

1. Did he learn anything from using TRAPS?
  2. What did he gain from using TRAPS?
- Learned the correct translation strategy (for those who commit one or more

reversal errors); or

- Confirmed his correct translation approach (for those who does not commit any reversal error); or
- Adopted the '*trick*' just to get the correct answer (without understanding).  
As an example, from the series of questions he has answered, he knows the answers to similar questions asked later, or the correct equation to formulate is always the reversed of what he feels correct i.e the reversed equation.

3. Which type of explanation was more helpful or meaningful for him to remedy his misconceptions?

- the *Partial Canned Text* explanation; or
- the *Learner-TRAPS Belief* explanation

### 7.4.3 Methodology

In **Study 1**, the learners were engaged in TRAPS online session. They were requested to answer questions in the Pre-test session, Teaching & Learning session and ended with the Post-test session.

After they went through the online sessions, they were asked to give comments on TRAPS' interface:

1. Did they find it easy to understand and follow instructions in using TRAPS?
2. Did they feel comfortable answering TRAPS' questions?
3. Did they find it easy to understand the explanations in the remediation session?
4. Suggestions for improvement on any of the above matters.

As mentioned earlier, evaluation in Study 1 is to identify users' difficulties and concerns when interacting with TRAPS. Hence, we concentrate mainly on matters related to *interfacing with TRAPS* and are less concerned with the learners' performance in answering the relational algebra word problems.

The learners' comments<sup>2</sup> were taken into consideration. Necessary changes in terms of the interface and wording of questions were made in order to improve TRAPS-user interface.

This was then followed by **Study 2** of the first external evaluation. Again as in Study 1, the learners were requested to go through TRAPS online sessions. In all three sessions, the learners were allowed to use pencil and paper. A calculator was made available on the screen for the learner if they needed to use it in questions that required quantitative answers.

The learners worked through the three sessions in TRAPS. The interaction in these sessions were recorded<sup>3</sup> for all the learners. The record eased reference to online session interactions during data analysis. After the online sessions, the learners were verbally asked a list of questions and their responses were recorded.

The learners used the system under supervision. They engaged with TRAPS' online sessions for between twenty and thirty minutes. The evaluation was a one-to-one basis. The advantage of one-to-one testing has been described by (Gagne, Briggs, & Wager, 1988). He listed five advantages of a one-to-one testing, which allows an investigator to:

1. Make detail observations of how a learner interacts with the instructional materials being developed;
2. Observe the learner's capability;
3. Identify inappropriate expectations;
4. Detect unclear directions, questions and information;
5. Note unexpected features of the instructional situation.

A series of observable intermediate states, which were related to solving the questions in TRAPS, were recorded during the experiment. The intermediate states of reaction

---

<sup>2</sup> These will be described more in section 7.4.4.

<sup>3</sup> This was done by using the Unix 'script' command.

were in verbal<sup>4</sup> or written format.<sup>5</sup> This was to capture the intermediate reasoning process the learner goes through.

In addition to making such observations (as above) for each of the volunteers, assistance was given whenever the learners had difficulties with using TRAPS.

#### 7.4.4 Observation and results

In **Study 1** formative-external evaluation, the answers given by two subjects (ST and AD) on the questions asked are as follows:

1. Did they find it easy to understand and follow instructions in using TRAPS?

Both students found that interacting with TRAPS was easy. It is not a complicated interface and the instruction given is easy for them to follow.

2. Did they feel comfortable answering questions in TRAPS?

ST commented that she didn't like TRAPS not giving feedback in the first and last part of sessions i.e the pre-test and post-test. AD however did not find this uncomfortable. She had experienced using computerised educational packages that have a pre-test and post-test. She said, she somehow had guessed that the first and last part of TRAPS could be the pre and Post-tests.

Commenting on TRAPS teaching and learning session, both ST and AD said that at the beginning, they did not like the *no response* interface. This made them unsure of their answer and *nervous* in answering the following related question. But after they had answered several problem-sets, they learned TRAPS's feedback pattern, i.e feedback will be given after all questions related to one problem-set has been answered.

Other comments were made about the multiple-choice question format. AD liked this because she need only to type a single character. She also liked the message and opportunity provided to correct her answer when she typed an invalid character. ST suggested that she would prefer if TRAPS asked her for confirma-

---

<sup>4</sup> The learners expressed these verbally. Some of these expressions are quoted in the next section.

<sup>5</sup> The learner scribbled on the pieces of paper provided.

tion of the answer she gave. She said this would be helpful. ST commented on this because she had made a “slip error” by accidentally typing an incorrect (even though valid) character.

3. Did they find it easy to understand the explanation in the remediation session?

The LTB-explanation took longer time for both learners to understand. ST had suggested some changes of information arrangement in the explanation. She suggested that TRAPS should point out the questions she incorrectly answered before giving the reasons why her answers were incorrect.

AD only commented that it was hard for her to understand at first, but after finishing reading the whole explanation and looking back at the questions that TRAPS displayed again (those that she had incorrectly answered), she began to get the whole picture and understood the explanation given.

Both ST and AD found that the PCT-explanation was easy for them to understand if some of the terms such as *static comparison* was defined or explained in a simpler language.

We have taken into consideration all the feedback given by ST and AD. Regarding TRAPS’ low feedback approach, it is a teaching strategy that requires learners to think carefully before answering their questions. Such a teaching approach may be able to elicit entrenched misconceptions. In the pre and Post-tests, our aim is to evaluate the learner’s performance before and after the teaching and learning session. We purposely do not give any feedback to the user.

Changes of sequence of information had been made according to ST’s good suggestions. By displaying their incorrectly answered questions, it makes it easier for the learner to understand the LTB-explanation. Some changes of wording in English and less usage of technical terms had been made in the PCT-explanation text.

The above were several changes that were made before we proceeded to Study 2 formative-external evaluation.

A summary of the observations of the assessment of TRAPS by the five students in Study 2 evaluation is shown in Table 7.1. In discussing the learners’ performance, we

Student	Pretest miscon	TL session		Post test miscon	Approach used
		conflict	miscon		
1. OA	match keyword	✓	match keyword	no miscon	word order → operative
2. RM	no miscon	✓	coefficient	no miscon	operative
3. WY	variable	✓	variable, coefficient	no miscon	static comparison → ratio
4. EH	no miscon	×	no miscon	no miscon	ratio, operative
5. AU	variable	✓	variable	variable	static comparison → switching of variables

Note: miscon - is a short form for misconception

Table 7.1: Learners' performance when using TRAPS in Study 2 of the first formative-external evaluation.

refer to Table 7.1.

Learner OA matched the keywords in the problem statements given and mapped them directly to form the algebraic equation. He used the word order approach in the Pre-test and in the first few problem sets during the TL session. However, having gone through the remediation session he managed to drop his misconception on the word order approach and applied the operative strategy in the last few problem sets in the TL session and also in the Post-test.

Similar improvement was observed in learner WY. The difference between learner WY and learner OA is that WY applied the static comparison strategy. She confused the usage of variables in her Pre-test, and both the function of variables and coefficient during her TL session. Her Post-test performance showed that such misconceptions have disappeared. She has changed her incorrect strategy i.e from the static comparison



approach to the operative approach.<sup>6</sup>

An interesting observation on learner RM is that she performed well in her Pre-test applying the operative approach. However, she missed the last questions in problem set 6. This is due to the fact that the TL session consists of several provoking entrapment questions to elicit the learners deeply rooted misconceptions, if any. When TRAPS prompted that there's a contradiction of beliefs in problem set 6, learner RM thought silently and then exclaimed *"Oh! I know why... I was careless enough to agree with this statement. There ought to ... Okay, I was trapped here."* She was later more careful answering other questions in her TL and Post-test sessions. Learner RM used the operative approach through out her sessions on TRAPS. She is among the two learners in this first batch of subjects who are capable of translating relational algebraic problems correctly.

Learner EH is another learner who is capable of translating all the problem statements and questions in the Pre-test, TL session and Post-test. He applied both the ratio and operative strategies in all three sessions, very carefully.

Of the five subjects, learner AU seemed not to have gained much from her TL sessions in relation to improving her ability for translating relational algebra word problems. Similar to WY, she confused the usage of variables that led her apply the static comparison approach. During the experiment, she commented *"Hmm... in order for me not to commit the error again, I'll form the equation I think is correct (the reverse equation) and then switch the variables in the equation to get the one that TRAPS says correct equation. It is hard for me to accept that my equation (the reversed equation) is incorrect. I still feel comfortable with this (pointing to the reversed equation)"*. AU adopted the *switching of variables* approach. She did this without semantic understanding. This could be seen when she was easily trapped in some of the entrapment questions which require semantic understanding (as in the operative approach) when formulating the equation. TRAPS seemed not to be able to convince learners such as learner AU during the length of the experimental session. Perhaps a longer session with various sets of questions or other teaching approach such as simulation that shows

---

<sup>6</sup> This was observed within the same online session. It is not known if the misconceptions resurface after a long period, eg. another experiment on WY using TRAPS, three months later.

Subjects		Learner-TAPS Beliefs explanation (LTB-explanation)	Partial Canned Text explanation (PCT-explanation)	Prefer to refer to both explanations?
s t u d e n t	OA	understandable	more understandable	✓
	RM	understandable	understandable	✓
	WY	understandable	more understandable	✓
	EH	understandable	understandable	✓
	AU	understandable	more understandable	✓

Table 7.2: Learners’ feedback on the *LTB-explanation* and the *PCT-explanation* in Study 2 of the first formative-external evaluation.

the dynamic operations that the equation meant, would assist the learner to convince herself.

The learners were also asked **whether the LTB-explanation or PCT-explanation provided an explanation that they find easy to understand, and whether they prefer to have both types of explanations for reference.** Results of the learners’ feedback are summarised in Table 7.2. Both of the learners who performed well i.e. RM and EH felt they could understand the LTB-explanation just as well as the PCT-explanation. Learners OA, WY and AU however felt that PCT-explanation was more understandable for them. One possibility is that the LTB-explanation lacks cues in its sentences. As (Moore, 1995) pointed out, cues play a significant role in making discourse a more natural or human-like interaction. This could possibly be improved by adding suitable cues at the appropriate place in the explanation.

An interesting finding is noted that all five learners have a similar opinion i.e they prefer to have both the LTB and PCT explanations made available for them to refer to. Such a preference by all the learners suggests that there is a significant role of the LTB and PCT explanations for the learners to understand the reasons for their incorrect answers.

## 7.5 Second formative-external evaluation - Study 3

### 7.5.1 Subjects

Subjects involved in the evaluation were teachers who teach mathematics in high school. As suggested by (Mark & Greer, 1993), ITS could be appraised by independent human teachers which could be gained from their feedback on the system's strength and weaknesses and rated as to their adequacy. For that matter, our goal of getting the teachers' feedback in this evaluation is to find out if TRAPS' learner modelling does what it is supposed to do and how the learner modelling is assessed from an educator's perspective.

### 7.5.2 Material

The teachers were given hard copies of sample scripted online sessions. The scripted online sessions were taken from several of the learners' exercises in Study 2 of the first formative-external evaluation.

The evaluation conducted among teachers was meant to elicit the teachers' teaching strategy in this particular domain and whether or not they agree with TRAPS' derivation of beliefs. The handouts for the evaluation conducted with the teachers are given in Appendix C.

The issues we addressed with the teachers were:

1. How did the teachers' beliefs differ from those of TRAPS, regarding the learners' answers and TRAPS' interpretation of their answers?
2. How did the teachers rate the level of difficulty in the selected questions?
3. What did the teachers think of the two types of explanation generated for the learner's remediation session?
4. What did the teachers think of the system — its learner modelling and TRAPS as an educational assistant, etc. ?

### 7.5.3 Methodology

Appointments were made according to the teachers' convenience. The evaluation was conducted on a one to one basis with each teacher. We begin our evaluation with the teachers by describing our research background. We highlighted the terms and issues — relational algebra word problems, reversal error, misconceptions related to the reversed equation, and learners' *contradictory schemes*.

After describing these issues mentioned, six parts of a questionnaire were given to the teachers. Description of each the six parts of the questionnaire is as follows:

**Part I** - A sample of a TRAPS online session was given for the teachers to have an idea of the system when it is used by a learner.

**Part II** - Nine of TRAPS' word problems representing the concepts and misconceptions in relational algebra word problems were given. Each question addresses one or more specific concepts. Different concepts include **qualitative understanding**, **quantitative understanding**, **agree/disagree word order approach**, **concept of variable**, **concept of equal sign**, **concept of coefficient**, **agree/disagree on the reversed equation**, **agree/disagree on the correct equation**, **agree/disagree on the operative approach**, **formulation of equation** and **reversing the meaning of the problem statement**.

**Part III** - There were several word problems representing their various **levels of difficulty** in this section. The difficulty level is based on the conceptual understanding and reasoning that a learner may need when answering the question. The group of learners we refer to are those who know that the equation to be formulated from such relational algebra word problems is a multiplication or division equation, not an addition or subtraction equation. However, they tend to formulate the reverse of the correct equation.

The teachers were asked on how they would rate the level of difficulty for each question relative to each other, if given to the group of average-level learners in their class. They did so by indicating the difficulty level that suited each question.

The scale given was:

1 - easy          2 - moderate          3 - difficult

The teachers could also give their own comments, besides the difficulty level values.

**Part IV** - Samples of the *LTB-explanation* and *PCT-explanation* were enclosed. The teachers were requested to comment on both types of explanation by answering series of questions:

- Would they use any of the explanations?
- In what situation would each of the explanations be applied?
- Which of the two explanations is preferred?
- What other types of explanation would they use in their class?
- How does their teaching text book suggest they explain to learners with difficulties in translating relational algebra word problems?

**Part V** - A sample of TRAPS overall observation<sup>7</sup> on the learner's performance was enclosed. The teachers were asked if they found the information displayed in the metalevel observation useful for them to keep track of the learner's performance.

**Part VI** - The teachers could comment on anything related to TRAPS:

- The domain i.e relational algebra word problems;
- TRAPS' learner modelling;
- Possible application of TRAPS in other domains; and
- Their views on using such a computer based system as assistance in the real classroom.

All four teachers were given as much time as they needed to answer the questions. They completed the questionnaire at home since they need extra time to understand and follow the samples of online sessions before answering the questions asked.

---

<sup>7</sup> In cases where an observation conflict (Level- 2 conflict) occurs.

#### 7.5.4 Results

The information obtained from Study 3 external-formative evaluation is described by referring to the issues mentioned in section 7.6.2:

1. How did the teachers' beliefs differ from those of TRAPS, regarding the learners' answers and TRAPS' interpretation of their answers?

Referring to the data in Table 7.3, 90% of beliefs that TRAPS derived from the learner and its learner modelling were agreed with or similar to those that the teachers would derive. The other 10% which were disagreed with for the following possible reasons:

- The questions were ambiguous. The learner formulated an equation which should not be implied as a reversed equation if the variables have not been specified to represent which group in the problem (eg. C stands for number of children or C stands for the number of adults in the *kindergarten problem*);
- The learner's answer was based on common sense or his experience in the problem situation. Hence, he could have answered correctly but at the same time not understanding why his answer was correct;
- The learner could have made a slip such as a typing error (eg. typing c instead of d, with intention of answering d).

2. How did the teachers rate the level of difficulty in the selected questions?

The teachers' rating for level of difficulty in the questions were 70% similar to those of TRAPS' rating made in its system design. See Table 7.4. A difference of rating seemed to exist due to the following reasons:

- For questions with divisional equations, one of the teachers felt that learners were less likely to formulate a reversed equation. This contradicts our question design which was to make these questions harder i.e different from multiplication equations;
- Some of the questions could be easier if the learner had experienced the situation in the problem asked. So it could be easy for these learners and at the same time difficult for those who lacked experience;

Teachers	No. of beliefs agreed	No. of beliefs disagreed
SM	30	4
WA	32	2
AB	29	5
IH	32	2

Table 7.3: Teachers’ feedback on agreeing or disagreeing with the beliefs derived by TRAPS.

- Teacher IH is a new teacher. He has less experience with students who have difficulties learning math. He tended to base his rating on his own capability rather than a student’s capability;
  - The rating was done by the teachers and not the learners. A question could be found easy by a teacher but difficult by a learner.
3. What did the teachers think of the two types of explanation generated for the learner’s remediation session?

In terms of comparing the effectiveness and preference between two types explanation generated during the online sessions, Table 7.5 provides a summary of the results. We refer to the teachers’ feedback in Table 7.5 for the discussion below.

All four teachers commented that the LTB-explanation systematically shows the chain of reasoning which originate from the learner’s own answers which then lead to one of the conflicting beliefs that occurred. Three of the teachers would use such an explanation when tutoring a learner on a one-to-one basis. Hence, the LTB-explanation adds the characteristic of individualisation in TRAPS.

All four teachers have almost identical comments on the PCT-explanation, i.e its application and generality. The PCT-explanation would be used when they



Teachers	No. of similar rating in the level of difficulty	No. of different rating in the level of difficulty
SM	8	2
WA	8	2
AB	9	1
IH	6	4

Table 7.4: Teachers’ feedback on rating the level of difficulty of some questions in TRAPS.

teach a large group of learners because this explanation is more general and suits one-to-many teaching situations.

Another finding from this experiment is that three teachers and all learners agreed that both the LTB-explanation and PCT-explanation should be available in the remediation session. This will enhance the learners’ understanding in order to mend their misconceptions. One teacher (AB) said that he would only use PCT-explanation, because it is difficult for him to recall the sequence of logical steps involved in explaining using an LTB-explanation.

The LTB-explanation for Level 2 conflict is meant for the teachers’ reference. It is not shown to the learner. The LTB-explanation for Level 2 is generated and compiled in a separate file and could be accessed by the teacher only. All four teachers found that such an explanation is beneficial for them to keep track the learners’ record of performance.

4. What do the teachers think of the system — its learner modelling and TRAPS as an educational assistant?

It is encouraging to find that all four teachers are positive about using TRAPS as an assistant for students with translation difficulties and when misconceptions related to reversal error are not easily dislodged. The teachers felt that such a

Subjects		Learner-TAPS Beliefs explanation (LTB-explanation)	Partial Canned Text explanation * (PCT-explanation)	Will use both types of explanation?
teacher	SM	lev 1 - individual - systematic flow lev 2 - useful	for large group of learners	✓
	WA	lev 1 - individual - systematic flow lev 2 - beneficial	for general explanation	✓
	AB	lev 1 - logical sequence - difficult to recall lev 2 - useful	to use in any situation	X
	IH	lev 1 - small audience lev 2 - helpful	- for big class audience - for general description	✓

Note:  
 Lev 1 - Level-1 conflict  
 Lev 2 - Level-2 conflict  
 \* - only available at TAPS' Level-1 conflict

Table 7.5: Teachers' feedback on using the *LTB-explanation* and the *PCT-explanation* in classes. This was conducted in Study 3.

system could be useful for the students to practice on their own. Another reason is that is often difficult to locate resource material in this particular area when a student requires further exposure and understanding on this type of difficulties.

## 7.6 Summary of TRAPS' evaluation

The feedback received during the formative-external evaluation of TRAPS was generally very positive. Both the learners and teachers involved in our evaluation confirmed that translating relational algebra word problems was a confusing domain. The learners were enthusiastic about experimenting with the system. They commented that TRAPS is easy to use and a very useful assistance for those who need to improve their translation skills. The teachers enthusiasm were noticed when they look forward to proceed answering the questionnaires, after a short conversation briefing the research background were given to them.

One of the reasons for the evaluation of TRAPS was to identify its limitations, so that justified and accurate claims may be made about its capabilities. Based on these, further enhancements of the system can be made. In the evaluation of TRAPS there were several areas found where the system is limited and where improvement can be made.

TRAPS is obviously limited to a single pattern of relational algebra word problems. There are various relational algebra problems (in tabular and pictorial format) that cause confusion to many people. Some of the learners found that after going through five similar patterns of word problems they became a bit bored.

The following chapter will analyse the contribution made in this research to the field of AI in Education (AI-ED). The results of our evaluation will be used in the next chapter to highlight the achievements of this work, address further the limitations of TRAPS, and offer some suggestions for improvement and further research.

## Chapter 8

# Discussion and Further Work

In this chapter we discuss several issues. We begin with some comparison and similarities of other work that is closely related to ours. We then proceed with the analysis of the contribution that the research in this thesis has made, particularly in the AI-ED discipline. Directions for further or future related work are suggested at the end of this chapter.

### 8.1 Comparison with other related work

In this section, we will discuss two closely related pieces of work on contradictions i.e THEMIS (Kono et al., 1994) and AMMS (Paiva & Self, 1994). This is followed by a discussion on (Murray et al., 1990)'s physics tutoring system that is related to TRAPS in terms of dealing with entrenched misconceptions.

#### 8.1.1 On contradictions

The issue of maintaining consistent information in learner modelling has been raised among others by (Woolf & Murray, 1993), and (Self, 1992). A small but significant amount of research such as (Huang, 1993), (Paiva & Self, 1994) and (Kono et al., 1994) deals with maintaining consistency of information in learner modelling. The recent discussion of (Kono et al., 1994) on THEMIS seems to be among the most relevant to our work. THEMIS is a nonmonotonic and inductive model inference system which includes the Hypothetical Student Modelling Inference System (HSMIS).

This incorporates deKleer’s Assumption-based Truth Maintenance System (ATMS) to maintain consistency of the learner modelling process. Both TRAPS and THEMIS view learner modelling as inductive learning from sets of examples. TRAPS and THEMIS consider maintaining consistent information while modelling a learner as an important component in our systems. The issues which both systems consider are summarised in Table 8.1 and discussed as follows.

Issues in THEMIS	Related matters in TRAPS
1. 4 categories of contradiction: a. a learner changes his mind, causing inconsistency in his answers [A1] b. slips which caused oracle contradiction [A2] c. learner knowledge contradiction in his reasoning process [A3] d. assumption contradiction in modelling [B]	1. consideration taken as: a. resurfacing of misconceptions by learner. b. handled by repetition of isomorphic questions. c. captured in the triangle conflict d. captured in: i) triangle conflict ii) observation conflict.
2. single world and multi-world contradictions	2. contradiction occurs as: i) single world (between learner’s beliefs) and observation conflict (between TRAPS’ own beliefs) ii) multi-world (between learner’s and TRAPS beliefs)
3. formulate learner’s knowledge contradiction based on the concept discrimination structure and multi-world logic	3. formulate both learner’s & TRAPS’s knowledge contradiction based on BRILM’s meta-theory and TRAPS’ overlay learner modelling
4. calculates the certainty of each clause in its method level learner model by referring various information	4. calculates the question values that are related to the abstract properties of the questions which the learner has answered

Table 8.1: Some issues considered in THEMIS and TRAPS.

THEMIS classifies its student contradictions into 4 types. Each of these types are also considered in TRAPS. THEMIS’ A1 type of contradiction (when a student changes his mind and causes inconsistency in his answers) is what TRAPS considers as the resurfacing of misconceptions of the reversal error in relational algebra word problems.

TRAPS can diagnose this when an observation conflict occurs. In THEMIS, a student's answer is represented in an oracle. An oracle consists of a fact and its truth value. (Kono et al., 1994) refer to the student's slips which caused an oracle contradiction as A2. TRAPS tries to detect the student's slip errors by having isomorphic questions repeated and allowing him to view and change his answers after each set of questions. A contradiction in a student's reasoning process (THEMIS' A3) is captured in TRAPS' triangle conflict. However, (Kono et al., 1994) argue that this type of contradiction should not be resolved, but such contradictory knowledge should be represented as it is, for effective tutoring. Besides representing the learner's contradictory knowledge as it is, we find that it is useful to resolve the conflict in TRAPS. Resolving by revising the conflicting beliefs provides us with the justifications involved and the chain of reasoning that causes the contradiction. This information is then used as an explanation to the learner (for remediation) in the teaching session. Resolving the contradiction here also allows detection of a conflict between the same beliefs, if it arises again. This is relevant in particular for the resurfacing of misconceptions. In terms of THEMIS' assumption (type B) contradiction, TRAPS identifies and takes care of this in its triangle conflict, and also in the observation conflict.

THEMIS emphasizes its differentiation of the single world and multi world contradictions. This is due to its different approach of handling each dimension of contradiction. Both single and multi world contradictions occur in TRAPS too. The single world happens in the triangle conflict i.e between the learner's own beliefs, and in the observation conflict i.e between TRAPS' own beliefs. As for the multi world conflict, this arises in the triangle level conflict, i.e between the learner's and TRAPS' beliefs. TRAPS however does not differentiate the single and multi world contradiction since both dimensions of conflicts can be solved in similar ways by BRILM. Beliefs that belong to either the learner or the tutor (TRAPS) can be distinguished by BRILM's backchaining of the self-evident assumptions. These assumptions indicate whether it is the learner's or TRAPS' beliefs. Examples of BRILM's self-evident assumptions are *answers(student, X)*, and *says(TRAPS, Y)*, which indicates the believer of the belief proposition.

THEMIS formulates a student's knowledge contradiction based on the concept discrim-

ination structure and multi-world logic (Kono et al., 1994). TRAPS formulates the learner's knowledge contradiction by combining the overlay learner modelling method and BRILM's formal representation and deductive process. A similar issue for evaluating and calculating some values based on the learner's previous information arises during the process of resolving a contradiction. THEMIS' calculation refers to various sources of information such as the number of top-level traces that justify the clauses, whether the oracles of the clause are correct answers or not, and how old the oracles are (Kono et al., 1994). However, TRAPS calculation only refers to the abstract values of the questions the learner had answered and whether the questions were answered correctly or not. This is sufficient and feasible for TRAPS since it deals with a specific restricted domain. The features of the cascaded problem sets architecture which have been discussed, helps TRAPS to minimize the sources of information TRAPS needs to reference.

The discussion above has highlighted some similarities and different issues that arise when developing a learner model which utilises a reasoning maintenance system. THEMIS, being part of FITS (Framework for ITS) (Ikeda & Mizoguchi, 1994) is a complex and large system which is developed as a domain independent system. (Kono et al., 1994) have considered many fine grain aspects in diagnosing and resolving conflicting information in its student modelling. TRAPS on the other hand, concentrates on a small specific domain, relational algebra word problems. Its approach in the various issues above is believed to be adequate for TRAPS' research objective i.e to illustrate the potential of belief revision in a tutoring system for a domain where contradictions arise when learners commit the reversal error. A comparison in performance between THEMIS and TRAPS is not feasible currently. It remains to be seen which is more effective.

*However, should discuss scaling up problem*

Other relevant work is that of (Paiva & Self, 1994)'s *Agent Model Maintenance System* (AMMS). AMMS is a system that functions as an auxiliary tool to maintain learner models in accordance with the principles of system consistency and learner accuracy. AMMS presents reasons (endorsements) for the hypotheses created about the learner. These endorsements are based on the acquisition rules, stored in AMMS and referred to when choosing the most *trustable* learner model in its *trust function*.



When a learner interacts with an application system (an ITS or ILE, for example), the acquisition process in AMMS generates hypotheses that explain the learner's behaviour. The hypotheses are dependent on the actual state of the learner model. The acquisition rules (and mechanism) provide directions to make guesses for the choice of hypotheses. Some of these rules are stronger than others. When a contradiction of beliefs occurs, a choice for a stronger environment<sup>1</sup> has to be made. They may depend on the evidence of the action performed by the learner or on the domain of the interaction. A relation has been established between sets of endorsements and this relation represents the trust on the acquisition mechanisms. AMMS based on such a relation  $\ll_e$  and  $\ll_e$  to choose the stronger environment. A trust function in AMMS is defined as a function from a set of  $E$ -environments<sup>2</sup> into an  $E$ -environment  $y$  such that:  $y = \text{trust}(X)$  such that for all  $x \in X$   $y \ll_e x$  or  $y = x$ .

The notion of Paiva et. al's trust function shares to some extent the same notion of TRAPS' application of epistemic entrenchment. In revising conflicting beliefs based on epistemic entrenchment, the more entrenched beliefs are the ones more reliable or trusted<sup>3</sup> during the revision process. The difference arises when TRAPS backchains through the justifications involved, as it refers also to the justifications' numerical values i.e the entrenchment and question values. Paiva et. al rely more on their acquisition rules in application to their trust function. AMMS<sup>3</sup> revision process is a non-numeric approach, i.e it uses symbolic description.

Regarding the two principles in AMMS, i.e the system consistency and learner accuracy, such principles are also observed in TRAPS, i.e in the observation conflict and the triangle conflict, respectively.

<sup>1</sup> An environment is a set of assumptions. Two types of assumptions in AMMS: a) normal assumption - used for problem solving when making a decision; b) justified assumption - used to create hypotheses to explain a learner's behaviour. Justified assumptions are supported by the acquisition mechanism or endorsement nodes.

<sup>2</sup> Contain assumptions and the endorsements used to support some of the hypotheses.

<sup>3</sup> For our Level-1 (triangle) conflict, we based the decision on the assumption that TRAPS' beliefs are more trusted than those of the learners. For Level-2 (observation) conflict, we based on the evaluation of the question values. We have discussed these in section 6.6.

### 8.1.2 On entrenched misconceptions

Not many computer assisted educational systems address the issue of learners' entrenched misconceptions except that of (Murray et al., 1990). Murray et al. implemented the *bridging analogies* strategy in a tutoring system that remediates learners' conceptual difficulties in physics. These conceptual difficulties are due to the learners' tenacious misconceptions.

Bridging analogies, a teaching strategy developed by (Clement & Brown, 1984) is found to be appropriate to deal with entrenched misconceptions (Murray et al., 1990). This teaching approach uses the learner's correct intuitions (referred to as conceptual anchors) and applies to the learner's analogical reasoning in assisting him to extend his correct intuitions to the problem (referred to as target situation) where he has misconceptions. When a learner's performance indicates the existence of misconceptions, he is presented with an analogous situation which is intended to function as an anchor. If his answer on the anchor question is correct, he has exhibited contradictory answers for the target and anchor situations. This shows that the learner is unaware of the target and anchor situations as being analogous. The bridging analogies approach attempts to make the learner understand of the analogical relationship. This is done by presenting the learner a sequence of intermediate analogies, called bridging analogies. As (Murray et al., 1990) explain:

*"At some point (or points) the student should be faced with considering two situations for which he has given contradictory answers yet which he realizes are analogous. The cognitive conflict which results should motivate the student to change his mind about the misconceived situation."*

(Murray et al., 1990), p. 83

Murray et al.'s system and TRAPS' approach in dealing entrenched misconceptions share similar approaches in terms of:

- Applying low or delay feedback in system-learner interactions;
- Structuring the pre-defined questions (referred to as example situations in Murray

et al's system) based on each question's difficulty;<sup>4</sup>

- Presenting the questions to the learners with multiple choice answers;
- Confronting the learners with the contradict beliefs which results from his incorrect answers (for TRAPS) or inconsistent answers (for Murray et al's system).

However they differ in their pedagogical approach. Murray et al's system adapts the bridging analogies approach which aims to bridge the conceptual understanding gap between the two conflicting beliefs by using analogous examples. TRAPS' aim of confronting the conflicting beliefs is to highlight the reasoning of the justifications (which includes the misconceptions at some point) for each of the beliefs. These aims or sub-goals of both systems share the same overall goal i.e to motivate the learner to change his incorrect conceptual understanding when confronted with the cognitive conflict that arises and diagnosed in their reasoning.

## 8.2 Analysis of contribution

### 8.2.1 Modelling through cognitive conflict

The occurrence of contradictions of beliefs or information is usually regarded as a disadvantage to a modelling system, as described in (Huang, 1993), (Mizoguchi, 1993), and (Woolf & Murray, 1993). However, our research consider the occurrence of contradictory beliefs as an advantage for TRAPS' learner modelling technique. Detecting the occurrence of a belief conflict assists TRAPS to:

- Diagnose the contradictory schemes that exist in the learner's reasoning.
- Exhibit the characteristic of being entrenched in the learner's reasoning.

The former case refer to the Level-1 or triangle conflict while the latter to the Level-2 or observation conflict.

---

<sup>4</sup> However, the database structure for each system is different. TRAPS constructs its questions in a Cascaded Problem Set architecture, while Murray et. al construct theirs on a pre-defined network of example situations.

In systems that perform learner modelling with a reason maintenance system such as that of (Huang, 1993), (Kono et al., 1994) and (Paiva & Self, 1994), their resolving (on the contradict beliefs) process aims just to maintain the consistency of beliefs involved, and utilise only the “winner” (i.e one of the contradicted beliefs that remain in the belief set) when proceeding their systems’ interaction with the learner. TRAPS differs from these systems, where by it utilises not only the *winner* but also all the beliefs and justifications involved in the chain of reasoning that are involved in the conflict. These beliefs and justifications are used as the contents in TRAPS’ LTB-explanation.

Both processes in the reasoning maintenance system i.e detecting and resolving a contradiction are found to be useful for TRAPS’ diagnosing and remediation process. TRAPS has illustrated that *modelling through cognitive conflict* could be useful for a domain such as translating relational algebra word problems. It remains to be seen if such a modelling technique is applicable for other domains that share the characteristic of contradictions and entrenched misconceptions.

*Would you expect this to be a property of domains?*

## 8.2.2 Explanation

TRAPS generates its LTB-explanation from the learner’s and TRAPS’ beliefs.<sup>5</sup> By using the learner’s own beliefs as part of the explanation content, it allows an explanation to have the following features:

**Familiarity** - An explanation can be more understandable if the content consists of terms or beliefs familiar to the user.

**Convincing** - A convincing justification is one that is sound and logical, and where possible based on facts in which the learner believes.

Both features are among characteristics of understandable explanation discussed by (Kass & Finin, 1988). TRAPS generation of LTB-explanation demonstrate that the process of resolving a contradiction offers a good resource for obtaining beliefs or information that have the above characteristics of explanation content.

---

<sup>5</sup> Beliefs that cause the contradiction to occur.

LTB-explanation which exhibits the logical flow of the beliefs involved is also a mechanism to explain to the learner on *deep-level processing* in cognitive reasoning. Deep-level reasoning refers to (Laurillard, 1990)'s discussion of deep and surface level cognitive processes. As an example, if a learner is solving an arithmetic problem, the learner's blind observation of arithmetics rules (surface-level processing) may not mean that he understands why he does what he is doing (deep-level processing). TRAPS' LTB-explanation offers a type of explanation that assists a learner to understand the deep-level processing.

*well, maybe.*

### 8.2.3 Computational Mathetics

The scope of Computational Mathetics as discussed in (Self, 1991) among others includes:

- learning through cognitive conflict.
- learning from reasoning;
- learning from reflection;
- learning from knowledgeable agents;

Our recent discussion on modelling through cognitive conflict and on TRAPS' LTB-explanation seem to be relevant to the issues of learning through cognitive conflict and learning from reasoning. Our approach of assisting the learners to remediate their misconceptions by confronting them with their conflicting beliefs is a strategy that aims to make the learners reflect on their reasoning process. On the issue of learning from a knowledgeable agent, we limit our assumption that *TRAPS' (the tutor) beliefs or rather knowledge is always more reliable than that of the learners' belief*. In other words, we assume that TRAPS is more knowledgeable than the learner. This may be true in some learning environments (such as tutoring a focused domain in TRAPS) but not necessarily valid in other learning environments.

Further tests and analysis of our modelling technique, LTB-explanation, confrontation of contradicting beliefs, and on the reliability of the agents (learner, system or other

learner) should lead to some findings that may make some possible contribution to computational mathematics.

#### 8.2.4 Other applications of learner modelling

The learner modelling technique illustrated in this thesis is not only beneficial for the reversal error in algebraic word problems but may possibly contribute to learner modelling in several ways:

- modelling a learner's unstable performance i.e the system has to handle fluctuating learner's understanding or mastery levels;
- applied in other domains where student's misconceptions are resilient and tend to resurface;<sup>6</sup>
- relevant also for learners with poor performance in subject materials which he finds difficult to master.

### 8.3 Limitations

Currently, TRAPS is confined to the type of contradictions that relate to the reversal error, from TRAPS' point of view. We however do not consider contradictions that occur from the learners' point of view. This limitation is restricted by our assumption that TRAPS' knowledge is more reliable than that of the learner. To offer a more dynamic system-learner interaction, we have to go beyond this assumption. A knowledgeable agent in an ITS or ILE may not necessarily be the tutor or system. The learner himself may be a knowledgeable agent in some situations.

BRILM in TRAPS is a belief revision system that maintains its reasoning, based on justifications which are tagged with numerical values, i.e relying on preset beliefs' entrenchment values and question values. Assigning the appropriate numerical values has to be made carefully and well justified by relevant underlying concepts. The underlying concepts have to be obtained from analysis of the domain being dealt with. Such an

---

<sup>6</sup> This will be discussed again in section 8.4.3.



approach may be feasible for a limited and focused domain like translating relational algebra word problems into equations. It may not be an appropriate approach for an independent learner model that are to serve various types of domain. Non-numerical or symbolic approaches i.e rules and not depending on numerical values, such as those of (Kass, 1991), (Murray, 1991) and (Paiva & Self, 1994) are possible methods for a domain-independent learner modelling systems.

## 8.4 Directions for further research

### 8.4.1 Tutoring strategy

The bridging analogies tutoring approach as discussed earlier is another way of handling learners' entrenched misconceptions. Such an approach could be incorporated into TRAPS' instructional control flow. The combination of pedagogical method of TRAPS and (Murray et al., 1990)'s system may produce a more constructive tutoring pedagogy for systems meant for contradictions and entrenched misconceptions.

### 8.4.2 Other learning environments

As has been discussed in chapter 3, many students have difficulty with the semantics of algebraic notation which leads them to formulating the reversed equation in the *student-professor* and *Mindy's restaurant* type of algebra word problems. This difficulty to view equations as active operations on variable quantities rather than statement which describe a static scene can be assisted through computer programming. (Clement et al., 1980) shows that more students are able to formulate a correct equation when solving by writing a program. TRAPS can enhance its remediation session by providing a window for the learner to write a small program while going through TRAPS. The learner can get back to TRAPS normal session from the programming window. Such suggestions apply only for the learner who has a programming background.

Collaborative learning environments seems to be one of the current directions and interests in AI-ED systems. Collaboration between person and person, and person with system depends in part on models of another agent's beliefs, and how beliefs are revised. An environment for assessment must be able to formulate a justification for



a specific judgement. TRAPS' usage of BRILM could contribute in assisting such a modelling issue in a collaborative learning environment.

### 8.4.3 Other domains

As mentioned earlier in chapter 3, many people find algebra problems with relational propositions difficult to translate. This applies not only to

1. translating from words to algebraic equation; but also to
2. writing a sentence from a given two-variable linear equation;
3. forming an equation representing the relationship between two variables from a tabular format; and
4. producing an equation describing the relationship between two variables from a pictorial format.<sup>7</sup>

The most common error is to produce the reverse of the correct equation for cases 1, 3 and 4 and the reverse meaning of the sentence in case 2 as discussed in (Aziz, 1993). TRAPS currently deals with case 1 only. It will be interesting to extend TRAPS to handle cases 2, 3, and 4.

Besides relational algebra problems as mentioned above, several science domains hold the same characteristics of contradiction and entrenched misconceptions for some students and adults. Real world experiences contribute to intuitions and understanding about how the physical world works. Some of these intuitions and understanding are consistent with scientific laws or theories, while others contradict these laws. Intuitions (either correct or incorrect) which result from real world experience are not easily *taught away*. They are entrenched beliefs. Empirical studies on several science subjects such as relativity and quantum mechanics (Brewer & Chinn, 1991), biology (Wandersee, 1983), physics (Fredette & Lochhead, 1980), and statistics (Tversky & Kahneman, 1974) among others, have discovered the characteristics of contradictions

---

<sup>7</sup> Examples for each of these relational algebraic problems are in our survey questionnaire in Appendix B.

and entrenched misconceptions in the students' (those who have conceptual difficulties) reasoning.

It may be beneficial to apply TRAPS' learner modelling for such domains, individually.

Further analysis of the outcomes of these systems may provide significant findings which could lead to externalising contradictions and entrenched misconceptions learner modelling.

## 8.5 Conclusion

We have discussed in this chapter some similarities and differences between TRAPS and three other closely related i.e THEMIS (Kono et al., 1994), AMMS (Paiva & Self, 1994) and (Murray et al., 1990)'s physics tutoring system. This is followed by an analysis of contribution that our research offers to the area of AI-ED. Discussion on TRAPS' limitations and several possibilities for enhancements to overcome these limitations are described also. The last section of this chapter pointed to several interesting areas for further research.

The following chapter which is the final chapter of this thesis, offers a few concluding remarks.

## Chapter 9

# Conclusion

This thesis has demonstrated how an AI technique can be incorporated in learner modelling for contradictions and entrenched misconceptions.

The goal of this research, as laid down in chapter 1 was to to apply a belief revision system as the basis for learner modelling in a domain with contradictions and entrenched misconceptions (in particular translating relational algebraic word problems into equations) and explore the potential of such learner modelling. Thus, this thesis describes a tutoring system, TRAPS, which assists learners in remediating the conceptual difficulties that impede their skill in translating relational algebra word problems to mathematical symbols. BRILM, a belief revision system that revises the conflicting beliefs using the notion of *epistemic entrenchment* is a great assistance to TRAPS' learner modelling.

The different aspects of the system were described in individual chapters. Chapter 2 described the background of learner modelling and belief revision. We reviewed the domain of algebra word problems and the contradictory schemes that cause the reversal error in chapter 3. In chapter 4 we discussed the core of our learner modelling, i.e the belief revision system is made. In chapter 5 the learner modelling technique that we applied to handle entrenched misconceptions and contradiction was discussed. The discussion in this chapter continues with description of the implementation in chapter 6. In chapter 7 we discussed the formative evaluation of TRAPS as a prototype system and results of the evaluation. In chapter 8, the contribution of the research to the area of AI-ED was considered, with pointers to further work which could be carried out.

Various disciplines and previous work have been analysed and incorporated into the research of this thesis. Among the major ones are:

- Studies of the reversal error in translating relational algebra word problems into equations (Rosnick & Clement, 1981), (Clement, 1982) and (Lochhead & Mestre, 1988);
- Theories of how people behave towards entrenched and inconsistent information (Brewer & Chinn, 1991);
- Implementation of a Cascaded-problem-set teaching architecture (Schank, 1990);
- Theory of rational changes of belief (Gärdenfors, 1988);
- Formalism of a belief revision that combines the foundational and coherence approaches (Berendt & Smail, 1992);
- The application of belief systems for learner modelling (Self, 1988c) and
- Formal approaches to student modelling (Self, 1992).

The interdisciplinary nature of these has contributed to the development of BRILM and TRAPS.

As mentioned in section 1.4, we now refer to the detailed research aims that we attempted and summarise the achievement and limitations of our work:

1. The combination of *theories of how people deal with entrenched and contradicting beliefs* and TRAPS' instructional control has been made by the implementation of the *Cascaded-problem-set teaching architecture*. This has enabled TRAPS to elicit the learners' entrenched contradictory schemes. However, the limitation lies in TRAPS' preset questions and anticipated contradictory schemes.
2. The usage of BRILM to diagnose and resolve conflicting beliefs:
  - between the learner's beliefs and TRAPS' beliefs;
  - within the learner's own beliefs;
  - among TRAPS' own beliefs.

Diagnosing the first two categories of beliefs' conflict is feasible in TRAPS' Level-1 (triangle) conflict. Based on the assumption that TRAPS' beliefs are more reliable than that of the learner, we assigned TRAPS' beliefs to be more entrenched than the learner's beliefs. Resolving a conflict between the learners' beliefs and that of TRAPS is performed based on this assumption. Contradiction within the learner's own beliefs is evaluated by TRAPS and refers to TRAPS' beliefs also. Hence, the assumption mentioned above could be applied to reach consistency of beliefs. Detecting conflict of beliefs within TRAPS' own beliefs is made in the Level-2 (observation) conflict. Resolving it needs further evaluation of the  $Qv$  (question values) as described in detail in section 6.6. BRILM manages to resolve contradictions by the notion of epistemic entrenchment to gain consistency for the three categories of contradiction. However violation <sup>of</sup> the *principle of conservatism*<sup>1</sup> has been made to adapt BRILM for a tutoring environment.

3. The role of BRILM, in providing relevant beliefs and justifications as explanations to remediate the learners with conflicting and entrenched misconceptions, is exhibited in the LTB-explanation which TRAPS generates. It has been found by teachers that LTB-explanation is useful in one-to-one tutoring. The limitation is that the flow of sentences in the explanation is not presented in natural conversation. This may cause difficulty in understanding the explanation. Improvement on this can be made by adding cues and rephrasing the beliefs and justifications obtained from BRILM into more natural sentences.
4. Illustrate that TRAPS intends to lessen (or remove) learners' conceptual difficulties which impedes their skill in translating relational algebra word problems to mathematical symbols. TRAPS to some extent and for certain students that we have experimented with, does achieve the above aim. Results of the experiment in section 7.4.4 show that three out of four students (who commit the reversal error) manage to learn the correct approach of translation, and dropped their misconceptions of incorrect translation strategies. However, it is not known whether or not, and to what extent, these misconceptions will (in time) resurface. Another TRAPS' limitation is that it is unable to convince learners who

---

<sup>1</sup> This has been discussed in section 4.8.

do not learn the correct conceptual understanding, but who adopt *the trick* of switching of variables on the equation which he feels more comfortable with, i.e the reversed equation.


To conclude, the contribution of this thesis to the area of AI-ED can be summarised as follows:

- Modelling contradictions and entrenched misconceptions through cognitive conflict by using a belief revision system is feasible for some domains.
- Resolving contradiction of beliefs in a belief revision system is a practical resource for generating explanations that assist learners in understanding the deep-level processing of the problems that they are solving.
- Learning through cognitive conflict, learning from reasoning, learning from reflection and learning from knowledgeable agents are among the issues that this research has tapped. Further analysis may lead to some contribution for computational mathematics.
- This research is also applicable in a number of modelling scenarios such as — in situations where learners exhibit unstable performance, and subject areas in which many learners have resilient misconceptions, or <sup>be</sup>unaware of the occurrence of contradictory beliefs in their reasoning.

Thus, the research in this thesis which its investigation is made through TRAPS has contributed something to the field of AI-ED. TRAPS illustrates the use of a belief revision in modelling contradictions and entrenched misconceptions in the domain of translating relational algebra word problems, which has not been attempted before in the area of AI-ED.

It is hoped that this research could lead to other relevant work, in particular that which shares a general educational goal i.e to produce computer-based educational systems that are more effective and more relevant to the modern world.

# Bibliography

- Alchourrón, C., Gärdenfors, P., & Makinson, D. (1985). On the logic of theory change: Partial meet contract and revision functions. *The Journal of Symbolic Logic*, 50, 510–530.
- Anderson, J. (1983). *The architecture of cognition*. Harvard University Press, Cambridge, MA.
- Aziz, N. (1993). Translating algebraic problems: Using belief systems in modelling & mending students' misconceptions. DAI Discussion Paper 141, Dept. of Artificial Intelligence, University of Edinburgh, Scotland.
- Aziz, N., Pain, H., & Brna, P. (1995a). Handling contradictions in student modelling in Translating Algebraic Problems Systems (TAPS). In Jonassen, D., & McCalla, G. (Eds.), *Proceedings of the International Conference on Computer in Education (ICCE 95)*, pp. 614–621 Virginia. AACE. 
- Aziz, N., Pain, H., & Brna, P. (1995b). Modelling and mending student's misconceptions in translating algebra word problems using a belief revision system in TAPS. In Greer, J. (Ed.), *Proceedings of the 7th World Conference on Artificial Intelligence in Education (AI-ED 95)*, pp. 107–114 Virginia. AACE.
- Berendt, B. (1992). Computing belief revision. Master's thesis, Department of Artificial Intelligence, University of Edinburgh, Scotland.
- Berendt, B., & Smaill, A. (1992). Computing belief revision. DAI Research Paper 609, Department of Artificial Intelligence, University of Edinburgh, Scotland. Paper presented in the 2nd International Workshop on Logic Programming and Non-Monotonic Reasoning.
- Bernado, A., & Okagaki, L. (1994). Roles of symbolic knowledge and problem-information context in solving word problems. *Journal of Educational Psychology*, 86(2), 212–220.
- Bobrow, D. (1974). Natural language input for a computer problem solving system. Technical Report MAC-TR-1, Project MAC, MIT, Cambridge, Mass.
- Brewer, W. F., & Chinn, C. A. (1991). Entrenched beliefs, inconsistent information, and knowledge change. In Birnbaum, L. (Ed.), *The International Conference on the Learning Sciences - Proceedings of the 1991 Conference*, pp. 67–73 Charlottesville, Virginia. AACE.



- Brown, J., & Burton, R. (1978). Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science*, 2, 155-192.
- Brown, J., & VanLehn, K. (1980). Repair Theory: A generative theory of bugs in procedural skills. *Cognitive Science*, 4, 379-426.
- Bundy, A., Byrd, L., Luger, G., Mellish, C., Milne, R., & Palmer, M. (1979). Mecho : a program to solve mechanics problems. DAI Working Paper 50, Department of Artificial Intelligence, University of Edinburgh, Edinburgh, Scotland, UK.
- Burton, R. (1982). Diagnosing bugs in a simple procedural skill. In Sleeman, D., & Brown, J. (Eds.), *Intelligent Tutoring Systems*. Academic Press, London.
- Carpenter, T., Corbitt, M., Kepner, H., Lindquist, M., & Reys, R. (1980). Solving verbal problems: Results and implications for national assessment. *Arithmetic Teacher*, 28, 8-12.
- Cawsey, A. (1992). *Explanation and Interaction - The Computer Generation of Explanatory Dialogues*. MIT Press, Massachusetts.
- Cawsey, A., Galliers, J., Reece, S., & Jones, K. (1992). Automating the librarian: a fundamental approach using belief revision. Technical Report 243, Computer Laboratory, University of Cambridge, Cambridge, UK.
- Clancey, W., J. (1987). *Knowledge-Based Tutoring: The Guidon program*. MIT Press.
- Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education*, 13, 16-30.
- Clement, J., & Brown, D. (1984). Using analogical reasoning to deal with 'deep' misconceptions in physics. Technical Report 95, University of Massachusetts, Scientific Reasoning Research Institute, Amherst.
- Clement, J., Lochhead, J., & Monk, S. (1981). Translation difficulties in learning mathematics. *American Mathematical Monthly*, 286-290.
- Clement, J., Lochhead, J., & Soloway, E. (1979). Translating between symbol systems: Isolating a common difficulty in solving algebra word problems. Technical Report 79-19, Department of Computer and Information Sciences, University of Massachusetts, Amherst, Massachusetts.
- Clement, J., Lochhead, J., & Soloway, E. (1980). Positive effects of computer programming on students' understanding of variables and equations. Paper presented at National Conference of the Association for Computing Machinery.
- De Kleer, J. (1986). An assumption-based TMS. *Artificial Intelligence*, 28(2), 127-162.
- Derry, S., & Hawkes, L. (1993). Local cognitive modeling of problem-solving behaviour: an application of fuzzy theory. In Lajoie, S., & Derry, S. (Eds.), *Computers as Cognitive Tools*. Hillsdale, Erlbaum.
- Dillenbourg, P., & Self, J. (1992). A framework for learner modelling. *Journal of Interactive Learning Environments*, 2(2), 111-137.

- Doyle, J. (1979). A truth maintenance system. *Artificial Intelligence*, 12, 231–272.
- Doyle, J. (1983). The ins and outs of reason maintenance. Report CMU-C5-83-126, Department of Computer Science, Carnegie-Mellon University.
- Doyle, J. (1992). Reason maintenance and belief revision: Foundations vs. coherence theories. In Gärdenfors, P. (Ed.), *Belief Revision*, pp. 29–51. Cambridge University Press, Cambridge, U.K.
- Finin, T. (1989). GUMS - a general user modelling shell. In Kobsa, A., & Wahlster, W. (Eds.), *User Models in Dialog System*, pp. 411–440. Springer Verlag, Berlin.
- Fredette, N., & Lochhead, J. (1980). Student conceptions of simple circuits. *The Physics Teacher*, 18, 194–198.
- Fuller, G. (1977). *College Algebra*. D. van Nostrand, New York.
- Gagne, R., Briggs, L., & Wager, W. (1988). *Principles of instructional design*. Holt, Rinehart and Winston, New York.
- Galliers, J. (1992). Autonomous belief revision and communication. In Gärdenfors, P. (Ed.), *Belief Revision*, pp. 220–246. Cambridge University Press, Cambridge, U.K.
- Galliers, J. R. (1990). Belief revision and a theory of communication. Technical Report 193, University of Cambridge Computer Laboratory, University of Cambridge, UK.
- Gärdenfors, P. (1988). *Knowledge in Flux - Modeling the Dynamics of Epistemic States*. The MIT Press, Cambridge, U.K.
- Gärdenfors, P. (1992). Belief revision: An introduction. In Gärdenfors, P. (Ed.), *Belief Revision*, chap. 1, pp. 1–28. Cambridge University Press, Cambridge, U.K.
- Goldstein, I. P. (1982). The genetic graph: A representation for the evolution of procedural knowledge. In Sleeman, D., & Brown, S. (Eds.), *Intelligent Tutoring Systems*. Academic Press, London.
- Greeno, J. (1980). Some examples of cognitive task analysis with instructional implications. In Snow, R., Federico, P., & Montague, W. (Eds.), *Aptitude learning and instruction (Volume 2)*. Erlbaum, Hillsdale, New Jersey.
- Halff, H. (1988). Curriculum and instruction in automated tutors. In Polson, M., & Richardson, J. (Eds.), *Foundations of Intelligent Tutoring Systems*, pp. 79–108. Lawrence Erlbaum Associates, Hillsdale, New Jersey.
- Harman, G. (1986). *Change in View: Principles of Reasoning*. MIT Press, Cambridge, Mass.
- Hawkes, L., Derry, S., Diefenbach, T., & Kegelmann, H. (1993). A simple learning environment improves mathematical reasoning. In Nwana, H. (Ed.), *Mathematical Intelligent Learning Environments*, pp. 162–184. Intellect, Oxford, UK.

- Huang, X. (1993). Inconsistent beliefs, attention, and student modelling. *Journal of Artificial Intelligence in Education*, 3(4), 417-428.
- Huang, X., McCalla, G., & Neufeld, E. (1991). Using attention in belief revision. *AAAI*, 275-280.
- Ikeda, M., & Mizoguchi, R. (1994). FITS: A framework for ITS - a computational model of tutoring. *Journal of Artificial Intelligence in Education*, 5(3), 319-348.
- Kass, R. (1991). Building a user model implicitly from a cooperative advisory system. *User Modelling and User-Adapted Interaction*, 1(3).
- Kass, R., & Finin, T. (1988). The need for user models in generating expert system explanations. Technical Report MS-CIS-88-37, Dept. of Computer and Information Science, University of Pennsylvania, Philadelphia.
- Katz, S., Lesgold, A., Eggan, G., & Gordin, M. (1992). Modelling the student in SHERLOCK II. *Journal of Artificial Intelligence in Education*, 3, 495-518.
- Kono, Y., Ikeda, M., & Mizoguchi, R. (1994). THEMIS: A nonmonotonic inductive student modelling system. *Journal of Artificial Intelligence in Education*, 5(3), 371-413.
- Konolige, K. (1980). A first-order formalisation of knowledge and action for a multiagent planning system. Technical Note 232, SRI International, Menlo Park, California.
- Langley, P., & Ohlsson, S. (1984). Automated cognitive modeling. In *Proceedings of the American Association of Artificial Intelligence Conference*, pp. 193-197 Los Altos, CA. Morgan Kaufman.
- Laurillard, D. (1990). An innovative framework for intelligent tutoring systems. In Elsom-Cook, M. (Ed.), *Guided Discovery Tutoring: A Framework for ICAI Research*. Paul Chapman Publishing, London.
- LeBlanc, M. (1993). From natural language to mathematical representations: A model of 'Mathematical Reading'. In Nwana, H. (Ed.), *Mathematical Intelligent Learning Environments*, pp. 126-144. Intellect, Oxford, UK.
- Littman, D., & Soloway, E. (1988). Evaluation ITSs: The cognitive science perspective. In Polson, M., & Richardson, J. (Eds.), *Foundations of Intelligent Tutoring Systems*, pp. 209-242. Lawrence Erlbaum Associates, Hillsdale, New Jersey.
- Lochhead, J. (1983). Constructivist approaches to teaching mathematics and science at the college level. In *Proceedings of the Fifth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Vol. 2 Montreal.
- Lochhead, J., Eylon, B., Ikeda, H., & Kishor, N. (1985). Representation of mathematical relationships in four countries.. Paper presented at Annual Meeting of the Mathematical Associations of America.

- Lochhead, J., & Mestre, J. (1988). From words to algebra: Mending misconceptions. In Coxford, A., & Shulte, A. (Eds.), *The Ideas of Algebra, K-12 1988 Yearbook*. National Council of Teachers of Mathematics (NCTM), Virginia.
- Loftus, E., & Suppes, P. (1972). Structural variables that determine problem solving difficulty in computer-assisted instruction. *Journal of Educational Psychology*, 531-542.
- Mark, M., & Greer, J. (1993). Evaluation methodologies for intelligent tutoring systems. *Journal of Artificial Intelligence in Education*, 4(2/3), 129-153.
- Mayer, R. E. (1981). Frequency norms and structural analysis of algebra story problems into families, categories, and templates. *Instructional Science*, 135-175.
- Mayer, R. E. (1982). Memory for algebra story problems. *Journal of Educational Psychology*, 199-216.
- Mengel, S., & Lively, W. (1991). On the use of neural networks in intelligent tutoring systems. *Journal of Artificial Intelligence in Education*, 2, 43-56.
- Mestre, J., & Gerace, W. (1986). The interplay of linguistic factors in mathematical translation task. *Focus on Learning Problems in Mathematic (Winter)*, 8, 59-72.
- Mestre, J., Gerace, W., & Lochhead, J. (1982). The interdependence of language and translational math skills among bilingual hispanic engineering students. *Journal of Research in Science Teaching*, 19, 399-410.
- Mestre, J., & Lochhead, W. (1983). The variable-reversal error among five cultural groups. In Bergeron, J., & Nocolas, H. (Eds.), *Proceedings of the Fifth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME)*, pp. 180-88. PME.
- Mizoguchi, R. (1993). Student modelling in ITS. In *ICCE'93 - Tutorial Proceedings* Taipei, Taiwan.
- Moore, J. (1995). Discourse generation for instructional applications: Making computer-based tutors more like human. In Greer, J. (Ed.), *Artificial Intelligence in Education - Proceedings of the 7th World Conference on AI-Ed 95* Virginia. ACCE.
- Murray, T., Schultz, K., Brown, D., & Clement, J. (1990). An analogy-based computer tutor for remediating physics misconceptions. *Interactive Learning Environments*, 1(2), 79-101.
- Murray, W. (1991). An endorsement-based approach to student modelling for planner-controlled tutors. *IJCAI'91*, 1100-1106.
- Nathan, M. (1991). A simple learning environment improves mathematical reasoning. *Intelligent Tutoring Media*, 2(3/4), 101-111.
- Nathan, M. (1993). A simple learning environment improves mathematical reasoning. In Nwana, H. (Ed.), *Mathematical Intelligent Learning Environments*, pp. 162-184. Intellect, Oxford, UK.



- Ohlsson, S. (1993). Constraint-based student modelling. *Journal of Artificial Intelligence in Education*, 3(4), 429-447.
- Paige, J., & Simon, H. (1966). Cognitive processes in solving algebra word problems. In Kleinmuntz, B. (Ed.), *Problem Solving*, pp. 51-118. Wiley, New York.
- Paiva, A., & Self, J. (1994). A learner model reason maintenance system. In Cohn, A. (Ed.), *ECAI 94 - 11th European Conference on Artificial Intelligence*, pp. 193-196. John Wiley & Sons.
- Paiva, A., Self, J., & Hartley, R. (1994). On the dynamics of learner models. In Cohn, A. (Ed.), *ECAI 94 - 11th European Conference on Artificial Intelligence*, pp. 163-167. John Wiley & Sons.
- Paiva, A., Self, J., & Hartley, R. (1995). Externalising learner models. In Greer, J. (Ed.), *Proceedings of the 7th World Conference on Artificial Intelligence in Education (AI-ED 95)*, pp. 509-516 Virginia. AACE.
- Rosnick, P. (1981). Some misconceptions concerning the concept of variable. *Mathematics Teacher*, 74, 418-420.
- Rosnick, P., & Clement, J. (1981). Learning without understanding: The effects of tutoring strategies on algebra misconceptions. *Journal of Mathematical Behavior*, 3(1), 3-27.
- Schank, R., & Edelson, D. (1990). A role for AI in education: Using technology to reshape education. Technical Report 1, Northwestern University, Evanston, IL. Also appears in the *Journal of Artificial Intelligence in Education*, Winter 1990.
- Schank, R. C. (1990). Teaching architecture. Technical Report AI 3, Northwestern University Institute for the Learning Sciences, Evanston, IL 60201.
- Schauble, L. (1990). Belief revision in children: The role of prior knowledge and strategies for generating evidence. *Journal of Experimental Child Psychology*, 49, 31-57.
- Schoenfeld, A. (1985). *Mathematical problem solving*. Academic Press, San Diego, California.
- Self, J. (1988a). Bypassing the intractable problem in student modelling. In *Proceedings of ITS'88*, pp. 18-24 Montreal.
- Self, J. (1988b). Student model: What use are they? In P. Ercoli, & Lewis, R. (Eds.), *AI Tools in Education*, pp. 73-86. North-Holland, Amsterdam.
- Self, J. (1988c). The use of belief systems for student modeling. Invited paper that was presented at the 1st European Congress on Artificial Intelligence and Training.
- Self, J. (1990). Theoretical foundations for Intelligent Tutoring Systems. *Journal of Artificial Intelligence in Education*, 1(4), 3-14.
- Self, J. (1991). Computational mathematics: the missing link in Intelligent Tutoring Systems research? In Costa, E. (Ed.), *New Directions for Intelligent Tutoring Systems*. Springer-Verlag, Berlin.

- Self, J. (1992). Formal approaches to student modelling. In McCalla, G., & Greer, J. (Eds.), *Student Modelling*. Springer, Berlin.
- Self, J. (1993). Model-based cognitive diagnosis. *User Modeling and User-Adapted Interaction*, 3, 89–106.
- Self, J. (1995). The ebb and flow of student modelling. In Jonassen, D., & McCalla, G. (Eds.), *Proceedings of the International Conference on Computer in Education (ICCE 95)*, pp. 40a–40h Virginia. AACE.
- Sentance, S. (1993). *Recognising and responding to English article usage: an ICALL base approach*. Ph.D. thesis, Department of Artificial Intelligence, University of Edinburgh, Scotland. J7
- Shortliffe, E. H. (1976). *Computer Based Medical Consultations: MYCIN*. Elsevier North Holland Inc.
- Simon, H. A. (1980). Problem solving and education. In Tuma, D. T., & Reif, F. (Eds.), *Problem Solving and education: Issues in teaching and research*. Erlbaum, Hillsdale, N.J.
- Singley (1989). The algebra word problem tutor. In *Proceeding of AI and Education - Synthesis and Reflection*, p. 267 Amsterdam. IOS.
- Stanojevic, M., & Vranes, S. (1994). Using truth maintenance systems - a tutorial. *IEEE Expert*, 46–56.
- Tversky, A., & Kahneman, D. (1974). Judgement under uncertainty: Heuristics and biases. *Science*, 185, 1124–1131.
- Twidale, M. (1993). Redressing the balance: The advantages of informal evaluation techniques for Intelligent Learning Environment. *Journal of Artificial Intelligence in Education*, 4(2/3), 155–178. J
- Wandersee, J. (1983). Students' misconceptions about photosynthesis: A cross-age study. In *Proceedings of the International Seminar on Misconceptions in Science and Mathematics* Ithaca, NY. J
- Weaver III, C., & Kintsch, W. (1988). The conceptual structure of word algebra problems. Technical Report 88–11, Institute of Cognitive Science, University of Colorado, Boulder, Colorado.
- Woolf, B., & Murray, T. (1993). Using machine learning to advise a student model. *Journal of Artificial Intelligence in Education*, 3(4), 401–416.

## **Appendix A**

### **A sample of TRAPS online session**

#### **A.1 The Pre-test session**



```
*****
*
* Translating Relational Algebraic Problems System *
*                      TRAPS                      *
*
*****
```

Press "Return" to proceed ...

```
*****
```

We'll start with several questions first ...

```
*****
```

"There are 3 times as many children as adults at the playground."

Given that there are 4 adults, how many children are there in the playground?

- |       |      |
|-------|------|
| a. 3  | b. 4 |
| c. 12 | d. 7 |

Answer: c

Try represent the statement above in an equation using variables C and A.

- |                |                |
|----------------|----------------|
| a. $3C = A$    | b. $3A = C$    |
| c. $A = 3 + C$ | d. $C = 3 + A$ |

Answer: a

In the equation you formulated, " $3C = A$ "  
C stands for \_\_\_\_\_

- |             |                           |
|-------------|---------------------------|
| a. children | b. the number of children |
| c. adults   | d. child                  |

Answer: a

Kate told to you that,  
"There are 8 times as many sheep as cows in uncle Tom's farm."

Both you and Kate want to represent this in an equation.

Kate suggested that, the equation can be formulated directly from the order of keywords of the statement, and thus produce the equation as  $8S = C$ .

Do you think Kate is right?

- a. yes                      b. no

Answer: y

In the statement above, C stands for

- a. a cow                      b. the number of cows  
c. cows                      d. sheep

Answer: a

From the helicopter, the pilot could see that,

"There are 4 times as many ships as planes in the area."  
Do you think that there are more planes or ships that the pilot could see?

- a. planes                      b. ships

Answer: b

"The council has 9 more men than women in it".  
Use M for the number of men and W for the number of women to translate the given statement into an equation.

- a.  $9M = W$                       b.  $M = 9W$   
c.  $W + 9 = M$                       d.  $M = 9 + W$

Answer: a

Translate the statement below into an equation using E for the number of tourist in Edinburgh and L for the number of tourist in London.

"There are 3 times as many tourist in Edinburgh as in London."

- a.  $3L = E$                       b.  $3E = L$   
c.  $3 + L = E$                       d.  $L = 3 + E$

Answer: b

How do we show in an equation that there are more tourist in Edinburgh than in London?

- a. multiply 3 to E to make E more than L  
b. multiply 3 to L to make it as many as E

Answer: a

## A.2 The Teaching-Learning session

\*\*\*\*\*

Now, let us try translate some algebraic problems

\*\*\*\*\*

T: indicates - Input from TRAPS

L: indicates - Expecting input from the learner

Press "Return" to proceed ...

T: Represent the statement below in an algebraic equation:  
 "There are 5 times as many children as teachers in the kindergarten."  
 Use the letters C and T as variables in your equation teachers.

a.  $5C = T$     b.  $5T = C$

L: a

T: The statement "There are 5 times as many children as teachers in the kindergarten" tells us that there are more children than teachers.

yes/no (y/n)?

L: y

T: We can translate this statement by just following the order of the keywords (5, children, teachers) in the sentence given.

yes/no (y/n)?

L: y

T: In your translated equation, C and T each (respectively) stands for :

- a. children and teachers
- b. a child and a teacher
- c. the number of children and the number of teachers

L: a

××× CONTRADICTION DETECTED ×××

T: Aha ... there's a conflict here. You seem to believe:

there are more teachers than children  
 AND ALSO  
 there are more children than teachers  
 at the same time.

● maziah @ twain.dai:/hame/maziah/Prog/Exp

=====

The belief

there are more children than teachers  
and its justifications are correct

but its opposite belief,  
there are more teachers than children

and the reason it arises, due to

Your answer in this problem set -

1. C and T stand for children and teachers
2. can translate by following the order of the keywords in the sentence
3. the equation for the kindergarten problem is  $5C = T$

need to be fixed.

=====

NOTE: We will look at the next window for more explanation about it.  
While you are working in either of the windows, you may  
still browse or refer to the other window at the  
same time.

=> Press the <RETURN> key and follow the instruction below:

- => 1. Move the cursor to the next window,
- => 2. Type: m explain
- => and press the <RETURN> key, there.

>>> Conflict between TRAPS and your beliefs <<<

Let's look into the conflict between:

there are more teachers than children  
and  
there are more children than teachers  
which arises in this session.

The belief - there are more teachers than children,  
arises from:

1. Your answer -

C and T stand for children and teachers

which leads to  
confusing the variables with labels  
which leads to  
a reversal of the correct equation for the kindergarten problem  
which leads to  
there are more teachers than children  
which then leads to  
the opposite belief of there are more children than teachers

2. Your answer -

can translate by following the order of the keywords in the sentence

which leads to  
directly map the keywords from the sentence to an equation  
which then leads to  
a reversal of the correct equation for the kindergarten problem

3. Your answer -

the equation for the kindergarten problem is  $5C = T$

which then leads to  
a reversal of the correct equation for the kindergarten problem

=====

```
maziah @ twain.dai:/hame/maziah/Prog/Exp

=====

The belief - there are more children than teachers,
arises from:

1. Your answer -

    there are more children than teachers

2. According to TRAPS -

    there are more children than teachers

=====

=> Let's get back to the other window to continue by:
=> 1. Moving the cursor to the next window.
=> 2. Press the <RETURN> key, there.
```



Now, would you like to

- a. get more explanation on your incorrect answer(s)
- b. fix your answer(s)

L: b

>>> Fixing the incorrect answers <<<

T: Okay ... Let's get back to the question(s) where you have to fix the answer(s).

T: In your translated equation, C and T each (respectively) stands for :

- a. children and teachers
- b. a child and a teacher
- c. the number of children and the number of teachers

L: a

T: Ooops... you got it wrong again!  
Each variable does not refer to the physical object.  
It stands for a number, indicating the quantity of the object.  
C and T stand for the number of children and the number of teachers.

Press <RETURN> to continue ...

T: We can translate this statement by just following the order of the keywords (5, children, teachers) in the sentence given.

yes/no (y/n)?

L: n

T: Yes, you got it right this time. Well done!

Press <RETURN> to continue ...

T: Represent the statement below in an algebraic equation:  
"There are 5 times as many children as teachers in the kindergarten."  
Use the letters C and T as variables in your equation teachers.

- a.  $5C = T$
- b.  $5T = C$

L: b

T: That's right. Well done!

### A.3 The Post-test session

\*\*\*\*\*

Let's go through some last few questions ...

\*\*\*\*\*

"In 1992, there were 3 times as many road accidents in Glasgow as there were in Dundee," says the reporter.

Given that there were 21 road accidents in Dundee, how many accidents occurred in Glasgow?

- a. 21
- b. 7
- c. 3
- d. 63

Answer: d

Try represent the statement above in an equation using the variables G for the number of accidents in Glasgow and D for the number of accidents in Dundee.

- a.  $3G = D$
- b.  $G = 3D$
- c.  $G = 3 + D$
- d.  $D = 3 + G$

Answer: b

The doctor reported that,  
"There are twice as many virus X as virus Y in the patient's blood."

Both you and Kate want to represent this in an equation. Kate suggested that, the equation can be formulated directly from the order of keywords of the statement, and thus produce the equation  $2X = Y$ .

Do you agree with her?

- a. yes
- b. no

Answer: n

In the statement above, Y stands for

- a. virus Y
- b. the number of virus Y
- c. virus X
- d. the number of virus X

Answer: b

From the helicopter, the pilot could see that,

"There are 6 times as many planes as ships nearby the border."  
Do you think that there are more planes or ships that the  
the pilot could see?

- a. planes                      b. ships

Answer: a

"The village has 18 more fishermen than farmers in it".  
Use  $F_m$  for the number of farmers and  $F_s$  for the number of fishermen to  
translate the given statement into an equation.

- a.  $F_m = 18F_s$                       b.  $F_s = 18F_m$   
c.  $F_s = 18 + F_m$                       d.  $F_m = 18 + F_s$

Answer: b

Translate the statement below into an equation using  
 $P$  for the number of tourist in Paris and  $L$  for the  
number of tourist in London.

"There are 5 times as many tourist in London as there are  
in Paris this summer."

- a.  $5L = P$                       b.  $5P = L$   
c.  $P = L/5$                       d.  $L = P/5$

Answer: p

Oops ... Type a, b, c, or d : b

How do we show in an equation that there are more tourist  
in London than in Paris?

- a. multiply 5 to  $L$  to make  $L$  more than  $P$   
b. multiply 5 to  $P$  to make it as many as  $L$

Answer: b

## Appendix B

# Questionnaire for survey

### Procedure

Algebraic problems (in statement, equation, table and pictorial formats) will be given. Please answer these questions in the following steps:

1. Understand the given problem and describe the problem in your own words. Try to find numbers that state the relationship of the objects involved in the given problem.
2. Translate the given problem into an equation. However, for Problem 5 where a linear algebra is given, translate it to a statement.
3. Did you double check your answer? Yes/No. If yes, please indicate how you did this?  
Plug in numbers in the variables ☐  
Visualise by drawing pictures ☐  
Plot a graph ☐  
Other method ☐. Please indicate:
4. Did Step 3 make you modify your answer in Step 2? Yes/No.  
If yes, what is the equation (or statement for Problem 5)?
5. Indicate how confident you are with your final answer:  
Quite confident ☐  
Not that confident ☐  
Not confident at all ☐
6. You may give your personal remarks on the problem above.

### Problem 1

Write an equation using the variables G (for the number of goats) and C (for the number of cows) to represent the following statement:

*On a nearby farm, the number of goats is five times the number of cows."*

1. Understand the given problem and describe the problem in your own words. Try to find numbers that state the relationship of the objects involved in the given problem.

2. Translate the given problem into an equation.

3. Did you double check your answer? Yes/No. If yes, please indicate how you did this?

Plug in numbers in the variables [ ]

Visualise by drawing pictures [ ]

Plot a graph [ ]

Other method [ ]. Please indicate:

4. Did Step 3 make you modify your answer in Step 2? Yes/No.  
If yes, what is the equation?

5. Indicate how confident you are with your final answer:

Quite confident [ ]

Not that confident [ ]

Not confident at all [ ]

6. You may give your personal remarks on the problem above.

## Problem 2

Write an equation to represent the following statement:

*"There are 8 times as many people in China as there are in the UK".*

Let  $C$  be the number of population in China, and  $U$  as the number of population in the UK.

1. Understand the given problem and describe the problem in your own words. Try to find numbers that state the relationship of the objects involved in the given problem.

2. Translate the given problem into an equation.

3. Did you double check your answer? Yes/No. If yes, please indicate how you did this?

Plug in numbers in the variables ☐

Visualise by drawing pictures ☐

Plot a graph ☐

Other method ☐. Please indicate:

4. Did Step 3 make you modify your answer in Step 2? Yes/No.  
If yes, what is the equation?

5. Indicate how confident you are with your final answer:

Quite confident ☐

Not that confident ☐

Not confident at all ☐

6. You may give your personal remarks on the problem above.



### Problem 3

Write an equation to represent the statement below:

*"The fishing village has 9 more children than adults in it."*

Use C for the number of children and A for the number of adults in your equation.

1. Understand the given problem and describe the problem in your own words. Try to find numbers that state the relationship of the objects involved in the given problem.

2. Translate the given problem into an equation.

3. Did you double check your answer? Yes/No. If yes, please indicate how you did this?

Plug in numbers in the variables [ ]

Visualise by drawing pictures [ ]

Plot a graph [ ]

Other method [ ]. Please indicate:

4. Did Step 3 make you modify your answer in Step 2? Yes/No.  
If yes, what is the equation?

5. Indicate how confident you are with your final answer:

Quite confident [ ]

Not that confident [ ]

Not confident at all [ ]

6. You may give your personal remarks on the problem above.

#### Problem 4

Given the following statement:

*“At the newly opened restaurant, for every four people who ordered rice, there were five people who ordered noodles.”*

Translate the statement into an equation using R for the number of people who ordered rice and N for number of customers ordered noodles.

1. Understand the given problem and describe the problem in your own words. Try to find numbers that state the relationship of the objects involved in the given problem.

2. Translate the given problem into an equation.

3. Did you double check your answer? Yes/No. If yes, please indicate how you did this?

Plug in numbers in the variables [ ]

Visualise by drawing pictures [ ]

Plot a graph [ ]

Other method [ ]. Please indicate:

4. Did Step 3 make you modify your answer in Step 2? Yes/No.  
If yes, what is the equation?

5. Indicate how confident you are with your final answer:

Quite confident [ ]

Not that confident [ ]

Not confident at all [ ]

6. You may give your personal remarks on the problem above.

### Problem 5

Write a statement that gives the same information as the following equation:

$$A = 3S$$

A in the equation representing the number of assemblers in a factory, while S represents the number of solders.

1. Understand the given problem and describe the problem in your own words. Try to find numbers that state the relationship of the objects involved in the given problem.

2. Translate the given linear equation into a statement.

3. Did you double check your answer? Yes/No. If yes, please indicate how you did this?

Plug in numbers in the variables [ ]

Visualise by drawing pictures [ ]

Plot a graph [ ]

Other method [ ]. Please indicate:

4. Did Step 3 make you modify your answer in Step 2? Yes/No.  
If yes, what is the statement:

5. Indicate how confident you are with your final answer:

Quite confident [ ]

Not that confident [ ]

Not confident at all [ ]

6. You may give your personal remarks on the problem above.

**Problem 6**

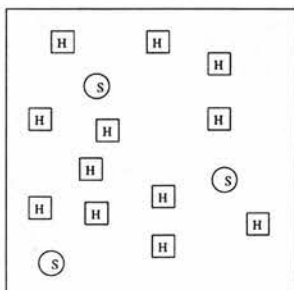
Weights are a spring the stretch of the spring is measured. The data is shown as below:

Stretch (cm)	Weight (g)
3	100
6	200
9	300
12	400

1. Understand the given problem and describe the problem in your own words. Try to find numbers that state the relationship of the objects involved in the given problem.
2. Formulate an equation from the given table.
3. Did you double check your answer? Yes/No. If yes, please indicate how you did this?  
Plug in numbers in the variables ☐  
Visualise by drawing pictures ☐  
Plot a graph ☐  
Other method ☐. Please indicate:
4. Did Step 3 make you modify your answer in Step 2? Yes/No.  
If yes, what is the equation?
5. Indicate how confident you are with your final answer:  
Quite confident ☐  
Not that confident ☐  
Not confident at all ☐
6. You may give your personal remarks on the problem above.

### Problem 7

From the picture below, write an equation using the letter H and S to describe the relationship between the number of housing estates (H) and the number of schools (S) in a city. Assume that H and S is directly propotional. The equation should allow you to calculate the number of schools given the number of housing estates.



1. Understand the given problem and describe the problem in your own words. Try to find numbers that state the relationship of the objects involved in the given problem.
2. Formulate an equation from the picture above.
3. Did you double check your answer? Yes/No. If yes, please indicate how you did this?  
Plug in numbers in the variables ☐  
Visualise by drawing pictures ☐  
Plot a graph ☐  
Other method ☐. Please indicate:
4. Did Step 3 make you modify your answer in Step 2? Yes/No.  
If yes, what is the equation?
5. Indicate how confident you are with your final answer:  
Quite confident ☐  
Not that confident ☐  
Not confident at all ☐
6. You may give your personal remarks on the problem above.

## Appendix C

# Questionnaire for Study 3

### Research background

Several empirical studies have observed that many students face difficulties in translating algebra word problems with sentences that state relationships between two variables, e.g. (Mayer, 1982) and (Paige & Simon, 1966). Among the typical errors that are often made in translating such problems is formulating a *reversed equation*, as discussed by (Lochhead & Mestre, 1988) and (Rosnick, 1981). This error is referred to as the *reversal error*. It has been observed that some of the students' misconceptions which lead to the reversal error are caused by inconsistency and contradiction of their beliefs in reasoning.

Let's look at a classic example of a relational algebra word problem, '*There are 6 times as many students as professors in this university.*' Try to translate this statement into an algebra equation.

Experiments with this problem had been carried out among engineering undergraduate students. Results were that 37% of them answered incorrectly and 68% of the incorrect answers were the reverse equation i.e  $6S = P$ . (Clement, 1982).

Using the above example and the analysis of empirical experiments done by (Clement, 1982) and (Rosnick, 1981), in common cases the learner believes qualitatively that there are more students than professors. However, his formulated equation which is a reversed equation implies the opposite situation i.e there are more professors than students. We refer to such conflicts and misconceptions leading to the conflict as the learner's *contradictory schemes*.

This skill of translation is essential and crucial but teachers do not have much time to spend on it or to give full attention and assistance to students during the limited hours in class. Hence, there is a need to provide students with additional support in the form of an *Intelligent* computer-based Tutoring System (ITS), that will allow them to practice the translation process and hence improve their skill in translating relational word problems to algebraic equations. We have developed a prototype system in this domain of relational algebra word problems. The system is called TAPS or Translating Algebraic Problems System.

TAPS' approach to diagnose the existence of contradictory schemes in the learner's reasoning is by detecting any occurrence of conflicting beliefs. These beliefs comprise of:

1. the learner's beliefs (from his answers),
2. TAPS' interpretation of the learner's beliefs and
3. TAPS' own beliefs.

TAPS keeps a record of all these beliefs when it interacts with the learner.

When a conflict of two beliefs has been detected, it then resolves the contradictory beliefs by choosing one of the two conflicting beliefs. Choosing the correct belief is based on the validity of the belief and its justifications. Resolving the conflict is part of TAPS' approach to assist the learner in remediating his misconceptions that had lead to his contradictory schemes in his reasoning.

In an ITS, diagnostic and remediation process of the learner is referred to as the *learner modelling*.

We apply a *belief revision system* in TAPS' learner modelling. A belief revision system deals with contradictions of information (or beliefs) in the light of new evidence. The system then resolves the contradiction between beliefs to maintain the consistency of beliefs in a knowledge base.

The application of a belief revision system in TAPS is our attempt to illustrate our learner modelling approach for the domain of translating relational algebra word problems.

Our goal of getting your valuable feedback in this evaluation is to find out if TAPS' learner modelling does what it is supposed to do and how the learner modelling is assessed from the educators perspective.



## Part I

A sample of TAPS online session is given for you to have an idea of the system when it is used by a learner.

While on TAPS, a learner is given several related questions for each set of problems. The number of questions in a set of problem varies and so does the difficulty level of the questions. A learner is given a chance to do as many sets of problem as he wants, after he finishes one.

'T:' stands for questions or information from TAPS, while 'L:' means the answer given by the learner. The questions are in multiple choice form. This is to make it easier for the learner to only enter a single character as their answer for each questions. This sample shows questions on problem set 1. After each question has been asked, the learner is expected to type in his answer. Immediate feedback of whether the answer is correct or incorrect is not given at this stage. TAPS proceeds with the next question and expects another answer from the learner. This is done until all the questions in the set of problem have been answered.

When the learner has answered all the questions in the problem set, TAPS will give its feedback on the answers. TAPS focuses on the occurrence or no occurrence of conflict between the learner's beliefs (his answers) and that of TAPS's beliefs (its knowledge i.e the correct answers and the underlying concepts for each answers). Therefore, the feedback given will be based on the **existence or nonexistence of contradictory beliefs**. If no contradiction has been detected, this means the questions have been answered correctly (i.e the learner does not exhibit contradictory schemes in his reasoning for this particular problem set). In cases where a contradiction of beliefs has been detected (which in this example, it has), TAPS will point this out to the learner. This is then followed by details about which of the two conflicting beliefs is correct ( $B_1$ ) and incorrect ( $B_2$ ), and which of the learner's answers relate to the incorrect belief.

The learner is then given a choice of either getting an explanation or fixing his answers. If an explanation is needed, two types of explanation will be given. The first type, referred to as the *LTB-explanation* consists of the Learner's and TAPS' beliefs that justify how ( $B_2$ ) and ( $B_1$ ) are derived.

Having read this explanation, the learner is again given another choice of either getting a further explanation or to amend his incorrect answers. If he wants further explanation, another type of explanation called the *PCT-explanation* (Partial canned-text explanation) will be displayed. This explanation explains to the learner in terms of the incorrect strategy (and misconceptions leading to it) which the learner has used.

When the learner has gone through these explanations or skipped one or both of the explanation parts, he can then change his answers. TAPS will display the questions that has been answered incorrectly for the learner to answer again.

## Part II

This section consists of several TAPS' relational algebra word problems. Each question has its own one or more specific concepts. Different concepts include qualitative understanding, quantitative understanding, agree/disagree word order approach,<sup>1</sup> concept of variable, concept of equal sign, concept of coefficient, agree/disagree on the reverse equation, agree/disagree on the correct equation, agree/disagree on the operative approach,<sup>2</sup> formulation of equation and reversing the meaning of the problem statement.

Example:

In the equation  $6S = P$ , the letter S stands for \_\_\_\_\_.

a. students

b. the number of students

The above question tries to test the learner's understanding on the concept of variable in an equation.

The learner's answer for each questions are taken as his beliefs by TAPS. TAPS then interpretes this belief further to other beliefs.

In each of them, the learner's beliefs (from his answer)<sup>3</sup> and TAPS' interpretation of the beliefs are shown in italics. We would like to get feedback of what you think about the beliefs involved in each questions. You may answer by filling in the given partial statements or make your own comments.

Below is an example that will be done together with the researcher.

- 
1. Represent the statement below in an algebraic equation:

"There are 5 times as many children as teachers in the kindergarten."

Use the letters C and T as variables in your equation.

a.  $5C = T$

b.  $5T = C$

which meaning?

The learner's belief:

If the learner answers a: The learner believes that *'the equation for the kindergarten problem is  $5C = T$ .'*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

---

<sup>1</sup> The student maps directly from the order of the keywords in the problem statement. This is done syntactically without understanding the meaning of the equation formulated.

<sup>2</sup> The student views the equation as representing an operation on a variable quantity to produce a number equal to another unspecified quantity.

<sup>3</sup> not considering occurrence of slip errors

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'the equation for the kindergarten problem is  $5C = T$ '*  
suggests that  
*'a reversal of the correct equation ( $5T = C$ ) for the kindergarten problem'*  
which leads to  
*'there are more teachers than children, according to the standard algebra interpretation'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **b**: The learner believes that *'the equation for the kindergarten problem is  $5T = C$ '*.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'the equation for the kindergarten problem is  $5T = C$ '*  
which leads to  
*'the learner formulates the correct equation for the kindergarten problem'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

2. In your translated equation, C and T each (respectively)  
stands for :

a. children and teachers

b. the number of children and the number of teachers

The learner's belief:

If the learner answers a: The learner believes that '*C and T stand for children and teachers*'.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'C and T stand for children and teachers'*

suggests that

*'the learner confuses the variables with labels'*

which leads to

*'a reversal of the correct equation for the kindergarten problem'*

which leads to

*'there are more teachers than children, according to the standard algebra interpretation'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **b**: *The learner believes that 'C and T stand for the number of children and the number of teachers'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'C and T stand for the number of children and the number of teachers'*  
suggests that  
*'the learner knows that the variables stand for unspecified number, not labels'*  
which leads to  
*'the learner has correct understanding on the concept of variable'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

3. "There are 7 times as many children as adults at the playground."  
Given that there are 14 adults, how many children are

there at the playground

- a. 2                                      b. 14  
c. 98

The learner's belief:

If the learner answers **a:** The learner believes that *'when there are 14 adults there are 2 children at the playground'*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'when there are 14 adults there are 2 children at the playground'*

leads to

*'there are more adults than children'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **b:** The learner believes that *'when there are 14 adults there are 14 children at the playground'*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'when there are 14 adults there are 14 children at the playground'*  
suggests that  
*'the learner has difficulty in quantitative and qualitative understanding'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **c**: The learner believes that *'when there are 14 adults there are 98 children at the playground'*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'when there are 14 adults there are 98 children at the playground'*  
leads to  
*'there are more children than adults'*



I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

4. To make the number of children (C) more than the number of adults (A), we should multiply C by 7, so that C will be more than A.  
Yes/No (y/n)?

The learner's belief:

If the learner answers y: The learner believes that '*multiply C by 7, so that C will be more*'

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'multiply C by 7, so that C will be more'*

leads to

*'the coefficient 7 is being associated with C to make C as the larger group'*

which leads to

*'misconception on the function of the coefficient (7) and the variables (C and A)'*

which leads to

*'a reversal of the correct equation for the playground problem'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **n**: The learner believes that *'it is incorrect to multiply C by 7, in order for C to be more than A'*.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'it is incorrect to multiply 7 to C, in order for C to be more than A'*  
suggests that  
*'the learner has correct understanding on the concept of coefficient and variable'*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

5. Supposed Kate stated to you that,  
"There are 8 times as many sheep as cows in my uncle's farm."

Both you and Kate want to represent this in an equation.  
Kate suggested that the equation can be formulated directly  
from the order of keywords of her statement, and thus produce  
the equation as  $8S = C$ .

Do you think Kate is right?      Yes/No      (y/n)?

The learner's belief:

If the learner answers **y**: The learner believes that *'the farm-equation can be formulated directly from the order of the problem statement'*.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'the farm-equation can be formulated directly from the order of the problem statement'*  
suggests that

*'the learner agrees with the word order approach'*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **n**: The learner believes that *'the farm-equation can not be formulated directly from the order of the problem statement'*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'the farm-equation cannot be formulated directly from the order of the problem statement'*

suggests that

*'the learner rejects word order approach'*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

6. During your visit to the University of Edinburgh, the tour leader told you that,  
"There are 6 times as many students as professors in this university."

Let  $S$  be the number of students and  $P$  the number of professors.

Do you find that the equation above should be  $6S = P$  instead of  $6P = S$  ?

- a. Yes,  $6S = P$  is the correct equation
- b. No,  $6P = S$  should be the correct equation

The learner's belief:

If the learner answers **a**: The learner believes that ' $6S = P$  is the correct equation for the student-professor problem instead of  $6P = S$ '.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*' $6S = P$  is the correct equation for the student-professor problem instead of  $6P = S$ ' suggests that  
'the learner rejects the correct equation for the student-professor problem'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **b**: The learner believes that *' $6P = S$  should be the correct equation for the student-professor problem'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*' $6P = S$  should be the correct equation for the student-professor problem'*

suggests that

*'the learner accepts the correct equation for the student-professor problem'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

7. From the helicopter, the pilot could see that,  
"There are 4 times as many ships as planes in the area."

Sam says that there are 100 planes when there are 25 ships.  
He gets this by plugging in 25 in S in his formulated  
equation  $4S = P$ , with S being the number of ships and P  
being the number of planes.

Does Sam uses the correct formulated equation?

- a. yes,  $4S = P$  is the equation  
b. no,  $4P = S$  is the correct one

The learner's belief:

If the learner answers a: The learner believes that *' $4S = P$  is the correct equation for ship-plane problem'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*' $4P = S$  is the correct equation for the ship-plane problem'*  
suggests that

*'the learner rejects the reverse equation and formulate the correct equation for the ship-plane problem'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **b**: The learner believes that *' $4P = S$  is the correct equation for the ship-plane problem'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*' $4S = P$  is the correct equation for ship-plane problem'*

suggests that

*'the learner accepts the reverse equation as the correct equation for the ship-plane problem'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_



Other comments: \_\_\_\_\_

8. Jean wants to prepare a home-made salad dressing.  
According to the recipe, she has to mix 3 times as  
much oil as vinegar.  
Does Jean have to put more oil or vinegar?  
a. vinegar      b. oil

The learner's belief:

If the learner answers **a**: The learner believes that '*Jean has to mix more vinegar than oil*'.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'Jean has to mix more vinegar than oil'*

leads to

*'there are more vinegar than oil'*

which leads to

*'the learner has an incorrect qualitative understanding'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **b**: The learner believes that '*Jean has to mix more oil than vinegar*'.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'Jean has to mix more oil than vinegar'*

leads to

*'there are more oil than vinegar'*

suggests that

*'the learner has correct qualitative understanding'*.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

9. According to the equation  $3V = L$ , if Jean has put a cup of oil, she will have to mix it with 3 cups of vinegar.

Do you agree with that? Yes/No (y/n) ?

The learner's belief:

If the learner answers **y**: The learner believes that '*for 1 cup of oil mix 3 cups of vinegar is correct*'.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'for 1 cup of oil mix 3 cups of vinegar is correct'*

leads to

*'mix more vinegar than oil'*

suggests that

*'the learner reverses the meaning of the problem statement'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

The learner's belief:

If the learner answers **n**: The learner believes that *'for 1 cup of oil mix 3 cups of vinegar is incorrect'*.

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

Other comments: \_\_\_\_\_

TAPS' implication on the learner's beliefs:

*'for 1 cup of oil mix 3 cups of vinegar is incorrect'*

leads to

*'mix more oil than vinegar'*

suggests that

*'the learner is not reversing the meaning of the problem statement'.*

I disagree because \_\_\_\_\_

I agree \_\_\_\_\_

I don't understand \_\_\_\_\_

I would imply the learner's belief as \_\_\_\_\_

Other comments: \_\_\_\_\_

### Part III

There are several word problems representing their various levels of difficulty in this section. The difficulty level is based on the conceptual understanding and reasoning that a learner may need when answering the question. The group of learners we refer to are those who know that the equation to be formulated from such relational algebra word problems is a multiplication or division equation, not an addition or subtraction equation. However, they tend to formulate the reverse of the correct equation. We refer to them as the *average-level* students.

How would you rate the level of difficulty for each question relative to each other, if given to the group of average-level students in your class?

Please tick the difficulty level which is in the following scale.

1 - easy      2 - moderate      3 - difficult

If you have any comments, you may write them in the space provided.

---

1. "There are 5 times as many children as teachers in the kindergarten."

We can translate this statement by just following the order of the keywords (5, children, teachers) in the sentence given.

Yes/No (y/n)?

Difficulty level:    1 - easy    2 - moderate    3 - difficult

Comments:

2. How can we show that there are more children when formulating the equation?

- a. multiply T by 5 to make it as many as C
- b. multiply C by 5 to make it the larger group

Difficulty level:    1 - easy    2 - moderate    3 - difficult

Comments:

3. How would you translate "There are 8 times as many sheep as cows in the farm" into an equation, using S as the number of sheep and C as the number of cows.

a.  $C = S/8$       b.  $S = C/8$

Difficulty level:    1 - easy    2 - moderate    3 - difficult

Comments:

4. During your visit to the University of Edinburgh, the tour leader told you that, "There are 6 times as many students as professors in this university."

Do you find that the equation above should be  $6S = P$  instead of  $6P = S$  ?

- a. Yes,  $6S = P$  is the correct equation  
b. No,  $6P = S$  should be the correct equation

Difficulty level:    1 - easy    2 - moderate    3 - difficult

Comments:

5. Sam says that there are 100 planes when there are 25 ships. He gets this by plugging in 25 in S in his formulated equation  $4S = P$ , with S being the number of ships and P being the number of planes.

Does Sam uses the correct formulated equation?

- a. yes,  $4S = P$  is the equation  
b. no,  $4P = S$  is the correct one

Difficulty level:    1 - easy    2 - moderate    3 - difficult

Comments:

6. During the Edinburgh Festival, "there are 4 times as many tourists in Edinburgh as there are in London".  
Represent this in an equation using X as the number of tourists in Edinburgh, and Y as those in London.

a.  $4Y = X$                       b.  $4X = Y$

Difficulty level:    1 - easy    2 - moderate    3 - difficult

Comments:

7. Referring to the statement "there are 4 times as many tourists in Edinburgh as there are in London",  
for a thousand tourists visiting Edinburgh, there  
are ----- tourist in London.

a. 4000                      b. 1000  
c. 40000                    d. 250

Difficulty level:    1 - easy    2 - moderate    3 - difficult

Comments:

8. Jean wants to prepare a home-made salad dressing.  
According to the recipe, she has to mix 3 times as  
much oil as vinegar.  
Does Jean has to put more oil or vinegar?

a. vinegar                      b. oil

Difficulty level:    1 - easy    2 - moderate    3 - difficult

Comments:

9. Next, try formulate an algebraic equation of the statement  
using the letter L for the amount of oil and V for the  
amount of vinegar.

a.  $3L = V$                       b.  $3V = L$

Difficulty level:    1 - easy    2 - moderate    3 - difficult



Comments:

10. From an equation  $3V = L$ , if Jean has put a cup of oil,  
she will have to mix it with 3 cups of vinegar.  
Do you agree with that? Yes/No (y/n) ?

Difficulty level: 1 - easy 2 - moderate 3 - difficult

Comments:

## Part IV

When remediating the learner's misconceptions, TAPS uses two types of explanation. They are:

1. an explanation which contains the learner's and TAPS' specific beliefs which are involved when the contradiction occurred. We refer to this as the **Learner and TAPS beliefs explanation** (*LTB-explanation*)
2. an explanation with contents that describes the translation strategy which the learner had applied and the correct translation approach that TAPS has. This type of explanation is referred to as the **Partial canned-text explanation** or the (*PCT-explanation*).

A sample of the *LTB-explanation* and *PCT-explanation* is enclosed. Please comment on both types of explanation by answering some of the questions below. You may add any other comments too.

---

1. Would you use the *LTB-explanation*?                      Yes ( )                      No ( )
2. If yes, in what situation would you use such an explanation? If no, why not?
3. How about using the *PCT-explanation*?                      Yes ( )                      No ( )
4. If yes, in what situation would you use such an explanation? If no, why not?
5. Do you think that the *LTB-explanation* and the *PCT-explanation* complement each other and are best used together?                      Yes ( )                      No ( )  
Further comments:
6. Or do you feel that one of the explanation is sufficient?                      Yes ( )                      No ( )
7. If yes, which explanation would you prefer? If no, why not?

8. What other type of explanation would you use in your class when some of your students have difficulty translating word algebra problems i.e they persist formulating the reverse equation instead of the correct equation?

9. How does the students' text book or the reference book you used for teaching students to formulate equation from relational algebra word problems explain to the students? Do you use the methods that the text suggests?

## Part V

When evaluating a learner's performance, there are times where a teacher at first believes that the learner has mastered the subject material (e.g concepts that are needed when formulating an algebra equation from a relational sentence). But from some new evidence (after several exercises), the teacher then discovers that the learner has not actually mastered the material yet. We consider this as another stage of conflict of beliefs i.e within the teacher's own beliefs.

In TAPS, such a situation can be traced. We show in this section a sample of TAPS' observation on two different problem sets. There is a contradiction of TAPS' beliefs from this observation. The beliefs involved are shown.

As a teacher, do you find the information in it useful for you to keep track of the learner's performance? And which particular information is most beneficial. If you have other views on this, you may write them below.

---

## Part VI

Any other comments regarding TAPS — its domain i.e relational algebra word problems, TAPS' learner modelling, application of TAPS in other domain, your views of using such a computer based system as your assistance in the real class room, or other matters can be written below.

---